John Phillips
1840

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A TREATISE OF GAUGING:

OR, THE Modern Practical Gauger.

CONTAINING,

Besides all the principal Rules usually given on the Subject, a great Variety of new and interesting IMPROVEMENTS: Particularly of gauging ALL SORTS of curvilineal Vessels, by the most EASY, CONCISE, and CERTAIN Method; which is NOW practised, and highly approved of, in and about this METROPOLIS.

WITH THE

DEMONSTRATIONS of several very useful and remarkable Properties of VESSELS and INSTRUMENTS, relative to this ART.

Illustrated with necessary EXAMPLES, and adapted both to the speculative and practical READER.

By THOMAS MOSS.

The SECOND EDITION:
Greatly ENLARGED and IMPROVED by the AUTHOR.

LONDON:
Printed for Z. STUART, and J. JOHNSON, Booksellers, in *Pater-noster Row.*
M.DCC.LXVIII.

BRITISH MUSEUM
31 JUL 51

TO THE

HONOURABLE

THE

COMMISSIONERS OF HIS MAJESTY'S REVENUE OF EXCISE, WITHIN THE KINGDOM OF *ENGLAND*, &c.

THE FOLLOWING

TREATISE,

Containing ſuch IMPROVEMENTS as will, it is humbly preſumed, contribute to the Service of the REVENUE, and alſo be of real Advantage to the practical GAUGER,

IS

MOST HUMBLY DEDICATED,

By their HONOURS *Faithful,* *

and moſt Obedient Servant,

Thomas Moſs.

* Faithful indeed! if ever man was so!

W. W.

THE

PREFACE.

THERE are ſo many different Compoſitions already extant on the Subject of Gauging, that Some, perhaps, may imagine no farther Improvements can poſſibly be made therein: How far ſuch an Opinion may be juſtifiable, is not my Province, in this Place, to determine: But, be that as it may, I flatter myſelf, however, that the attentive *and* unprejudiced *Reader will find ſeveral uſeful and important Things, in the following Sheets, not to be met with in any other Treatiſe on the Subject.*

In the Courſe of this Undertaking, I have exerted my utmoſt Endeavours to extend *the* ART *of* GAUGING; *by laying down the moſt general, accurate, and eaſy Methods of determining the Meaſures of all the various Forms of Veſſels which occur in Practice: In what Manner my Deſign is executed, is wholly ſubmitted to the* impartial *Reader.*

I have been ſtudiouſly anxious to promote Truth and Utility, without even attempting to depreciate the Labours of Others; as judging it more commendable to paſs over any little Imperfection that occurred, than to endeavour to magnify it, with a View to enhance the Merit of my own Performance: And although I have, in many Inſtances, departed from former Writers on the Subject, the greateſt Care has been taken not to introduce any new *Methods of Gauging, but ſuch as will, it is preſumed, be found, by the Practitioner, to be far more general, and not more difficult, than thoſe which are omitted; and alſo ſuch as are founded upon the moſt indubitable Principles: Which Principles, to oblige the inquiſitive Reader, are given in the* Notes, *and it is hoped they will not be decried by thoſe Gentlemen*

who

who may want Time, or Inclination, to apply themselves to the speculative Part of the Subject.

Though it must be allowed, that there is no Necessity for the practical Gauger to be acquainted with Geometry, Algebra, *and* Fluxions; *yet I can with Safety affirm (what has been already remarked, by that excellent Mathematician, the late Mr.* Robert Shirtcliffe, *in his Theory and Practice of Gauging)* " *That* " *without a competent Skill in Algebra and Geometry,* " *it is absolutely impossible for any Person to determine, whether the Rules given by common Writers* " *upon this Subject be true or false, and much less* " *make any (even the least) Improvement in this Part* " *of Science.*" — *And, therefore, I cannot but think it extremely absurd, and ill-natured in any One, to endeavour to depreciate, and, what is very astonishing, even to ridicule those excellent Branches of Science, from whence are derived the very* Rules *and* Instruments *which are so highly approved by every practical Gauger; without which, he must acknowledge, even his daily Business could not be performed.*

As it would be unnecessary, here, to enumerate all *the Particulars that compose the ensuing Sheets; it may therefore suffice to point out only a* few *of those Articles, which, perhaps, the candid and practical Reader will look upon as real and useful Improvements.*

In the Business of Cask-Gauging (which is reckoned the most difficult Part of the Subject) is given a general and practical Method of determining, very nearly, *the* true *Variety of any close Cask; whereby any Person, with very little Application, may be enabled to form a tolerable Idea of the Variety, by only* viewing *the Cask: This, it is presumed, is an Improvement, which, if duly attended to, will be found of singular Advantage; since it will, doubtless, be a*

Means

Means of preventing ſuch Errors as muſt unavoidably happen, by the ordinary Method of merely gueſſing *at the Variety of the Caſk. — In this Branch of Gauging are alſo given, two very eaſy and comprehenſive Methods of finding the true Mean-Diameters of the three different Varieties of Caſks, let the Proportion of the Bung and Head-Diameters be what it will: For on ſuch Proportion (and not upon the Difference of thoſe Diameters) the* true *Multiplier, for finding a Mean-Diameter, wholly depends.*

The Nature and Property of the Diagonal Rod are far more extenſively conſidered than heretofore; with very plain and uſeful Directions for applying this Inſtrument, with Certainty, to upwards of 100 *different Forms of Caſks.*

It has been hitherto imagined, that the Diagonal Rod would only exhibit the true *Contents of* one *particular Form of Caſks; and alſo that its original Conſtruction was from a Caſk, whoſe Diagonal is* 30 *Inches, and Content* 60 *Ale Gallons, or from ſome known Content and its correſponding Diagonal, as they appear on Gauging Rules.—That the Diagonal of a Caſk may be* 30 *Inches, and its Content* 60 *Ale Gallons (or about* $73\frac{1}{4}$ *Wine Gallons) is indiſputably evident: But it certainly does not follow from thence, that there can be but* one *Bung-Diameter, Head-Diameter, and Length, allotted for a Caſk, which can have the above-mentioned Diagonal and Content; becauſe the Proportion of* all *thoſe Dimenſions, and conſequently the Form of the Caſk, may vary; without altering either its* Diagonal, Magnitude, *or* Variety. *(See* Sect. X. Pa. 205*).*

The Methods of ullaging both ſtanding and lying Caſks, by the Pen, are given in as plain and conciſe a Manner as poſſible; with very eaſy Directions for determining when the Lines of Segments on the Sliding-Rule may be depended on, and alſo whether the Error is in Exceſs or Defect.

The

PREFACE.

The Method of approximating the Meaſure of any curvilineal Plane, by Means of equidiſtant perpendicular Ordinates (or Diameters), is delivered with as much Perſpicuity and Conciſeneſs, as the Nature of ſo important a Subject will poſſibly admit of; and which is moreover illuſtrated with Examples (ſuitable to the Practice of Gauging) not only of Figures whoſe Properties are known, and the Areas thereof determinable by other Methods; but alſo of Figures whoſe Properties are unknown, and their Areas not to be determined, with any Certainty, by any other Method whatever.

Very accurate Tables are given of the Areas of Circles in Ale and Wine Gallons, each to 216 Inches Diameter.—Tables of this Nature (though perhaps none ſo extenſive) are to be met with in moſt Authors on this Subject; but, however, the Methods of Computation (being more exact and eaſy than any that have occurred to me), by which the Tables in Pa. 262, &c. were actually formed, will not, it is apprehended, be unacceptable to ſuch Perſons as may be deſirous of either extending (if neceſſary) the ſaid Tables, or of examining the Truth of thoſe given by Others.

Many other uſeful and intereſting Particulars might here be mentioned, but I rather chooſe to refer to the Work itſelf; and therefore ſhall only beg, that the Reader will not too haſtily cenſure and condemn it; but that, after impartially peruſing it with proper Attention, he will candidly excuſe ſuch Defects as may occur to him, and have eſcaped my Obſervation: This, it is hoped, is no unreaſonable Requeſt; ſince it is but ſollicititing that Indulgence which every One is intitled to, who lays his OWN Sentiments before the Public, without ſhewing too high an Opinion of his own Abilities.

Bromley (Middleſex), January 5, 1768.

N. B. The firſt Edition of this Work was publiſhed in 1765.

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Lately

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I. In one Volume Octavo, Price 6*s.* bound,

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ERRATA.

Page 33, Line 4, for *and single*, read *and the single*.
P. 38, L. 6, for *Line*, r. *Lines*.
P. 40, L. 4, in the Note, for the *Semi-colon*, put a *Comma*.
P. 48, L. 1, in the Note, for *noted*, r. *denoted*.
P. 50, L. 12, for 25, r. 15.
P. 64, L. 18, for *preceeding*, r. *preceding*.
P. 84, L. 20, for 4924, r. 4624.
P. 92, L. 10, from the Bottom, dele the *Comma*; and for 1. r. .1.

P. 138, L. 5, from the Bottom, for $\overline{a-x}|^2$ (in some of the Copies), r. $\overline{d-x}|^2$.
P. 157, L. 11, in the Note, for *cef*, r. *ed*.
P. 193, L. 3, for *if*, r. *of*.
P. 210, L. 11, from the Bottom, for *oblate*, r. *oblong*.
P. 226, bottom Line, for $b \times b$, r. $b \times b$. And L. 9, from the Bottom, for $2b^2 - b^2$, r. $2b^2 + b^2$.
P. 230, L. 2, in *Prop.* 3, for *any Cask*, r. *any lying Cask*.
P. 231, L. 4, from the Bottom, instead of the *Period*, place a *Comma*.
P. 239, L. 19, for *and*, r. *of*.
P. 243, L. 3, from the Bottom, for 30.99, r. 30.96.
P. 250, L. 13, for BD, r. BS.

☞ The above *Errors* having escaped the Author's Observation till after the Sheets were printed off; the Reader is, therefore, desired to *remark them*, before he reads the ensuing Work.

Corrected.

A TREATISE OF GAUGING.

SECTION I.

Of DECIMAL FRACTIONS.

IT is indiſpenſably required of every One, who would learn the Art of Gauging, to acquire a previous and competent Knowledge of Decimal Fractions, as the Dimenſions of all Utenſils, of what Form ſoever, are taken in Inches and Tenths, and all Inſtruments, for that Purpoſe, are decimally divided. — I therefore apprehend it will not be improper to give, by Way of Introduction, a ſuccinct Account of Decimal Fractions.

Firſt then, in Order to form as clear an Idea as poſſible of the Nature of Fractions in general, let us conceive an Unit, Integer, or *one* whole Thing, of what Denomination ſoever, whether it be Coin,

Weight, Measure or Time, *&c.* to be divided into a certain Number of equal Parts; then this Number of equal Parts, be it what it will, is ever called the *Denominator* of the Fraction; and that Number shewing how many of these Parts are to be taken, or expressed, is called the *Numerator*.

Thus, for Example, suppose an Unit, or Integer, to be divided into 12 equal Parts, and that it was required to express 5 of those Parts in a Vulgar Fraction; then 12 will be the Denominator, and 5 the Numerator; and the Fraction itself, will, by writing the Numerator above the Denominator, with a Line drawn between them, be thus expressed $\frac{5}{12}$, and is read *Five-twelfths.* — This is a general Notation for Fractions of all Denominations.

But, in Decimal Fractions, where the *Integer*, or *Whole Thing*, is supposed to be divided into 10, 100, or 1000, *&c.* equal Parts, the Notation will be more commodious for Practice, by writing down the Number of Parts to be taken with a Point, or Comma, prefixed, without putting down the Denominator, as in Vulgar Fractions; it being absolutely unnecessary here, since it is always known to be an Unit, with as many Cyphers annexed, as there are Places in the Fraction taken. Thus $\frac{5}{10}$ (Five-tenths) expressed decimally, will be .5, and $\frac{75}{100}$ (Seventy-five Hundredths) will be .75.

It will be proper to observe to the Learner, that Cyphers placed on the Left-Hand of a Decimal Fraction, decrease its Value in a ten-fold Proportion; in the same Manner as Cyphers placed on the Right-Hand of a whole Number, increase the Value thereof.

For Example, .5 ($\frac{5}{10}$) is the Half of an Unit; but .05 ($\frac{5}{100}$) is only $\frac{1}{20}$th of an Unit, which is

$\frac{1}{10}$th of the former; alſo .005 ($\frac{5}{1000}$) is only $\frac{1}{200}$th Part of an Unit, and is therefore $\frac{1}{10}$th of the laſt Fraction .05.

It may alſo be proper to take Notice, that Cyphers placed on the Right-Hand of a Decimal Fraction, neither augment nor diminiſh its Value. For .5 (or Five-tenths) of any Thing, are the very ſame in Value as .50 (or Fifty-hundredths) of the ſame Thing: In the former of theſe, the Integer is ſuppoſed to be divided into 10 equal Parts, and in the latter into 100 equal Parts; hence it is very obvious, that 5 of the 10 equal Parts are equivalent to 50 of the 100 equal Parts, of the ſame *Integer*, or *Unit*.

	Hundreds of Thouſands.	*Tens of Thouſands.*	*Thouſands.*	*Hundreds.*	*Tens.*	*Unit's Place.*	*Tenths.*	*Hundredths.*	*Thouſandths.*	*Ten Thouſandths.*	*Hundred Thouſandths.*	
&c.	5	4	3	2	1	0.	1	2	3	4	5	&c.

In the preceding Table it is very obvious, that the Figures on the Left-Hand of the Point are Integers or whole Numbers; and that thoſe on the Right-Hand are Decimal Fractions.

PROP. I.

To reduce any given Vulgar Fraction, into a Decimal Fraction.

RULE.

Let a competent Number of Cyphers be annexed to the Numerator, to form a Dividend; which being divided by the Denominator, the Quotient (if there happens to be no Remainder) will be *precisely* the Decimal Fraction sought.*

EXAMPLES.

How must $\frac{3}{4}$ and $\frac{7}{8}$ of an Unit, or Integer, be expressed in Decimal Fractions?

OPERATIONS.

```
4)3.00(.75          8(7.000(.875
  28                  64
  ---                 ---
   20                  60
   20                  56
  ---                 ---
   ..                   40
  ---                   40
                       ---
                        ..
                       ---
```

Hence

* A Vulgar Fraction cannot *precisely* be expressed in a Decimal Fraction, unless the Denominator of the Vulgar Fraction is either some Power of 2, some Power of 5, or else some Power of 2 into some Power of 5; that is, universally, except the Denominator is $2^m \times 5^n$; supposing m and n to denote any whole Numbers whatever.

For it is very evident, that the Number 10 is divisible by none of the Digits, except 2 and 5; and it is proved (*Eu.* 8. *B.* 14. & 15. *Prop.*) if one Number measure another, that the Square (or Cube) of that Number will measure the Square (or Cube) of the other Number; from whence it follows, that any Power of the greater is divisible by the same Power of the less Number; and it is well known, that, in reducing any Vulgar Fraction into a Decimal Fraction, the Numerator of the former is ever multiplied (or supposed to be multiplied) by some Power of 10, and the Product divided by the

Hence it appears, that $\frac{3}{4}$ (Three-fourths) of an Unit, are equivalent to .75 ($\frac{75}{100}$), that is, to 75 Hundredth Parts of an Unit or Integer: Alſo, $\frac{7}{8}$ (Seven-eighths) of an Unit, are equal to .875 ($\frac{875}{1000}$), *i. e.* to 875 Thouſandth Parts of an Unit.

After the very ſame Manner, may any other Vulgar Fraction be reduced into a Decimal Fraction.

ADDITION *of* DECIMALS.

PROP. 2.

To find the Sum of any given Number of Decimal Fractions, or mixed Numbers.

The Method of Operation is the very ſame as in whole Numbers, ſtrict Regard being taken in placing the ſeparating Points one under another; and alſo to place Units under Units, *&c.* in the Integers or whole Numbers, and Tenths under Tenths, *&c.* in the Decimal Parts; and laſtly, to point off as many Decimals in the Total, as there are in that given Term which conſiſts of the greateſt Number of Decimal Places.

EXAMPLE.

Suppoſe it were required to find the Sum of the following mixed Numbers and Decimal Parts; *viz.*

26.489,

---

the Denominator: Hence it is plain, that unleſs the Denominator of the Vulgar Fraction be expreſſed by $2^m \times 5^n$ (*m* and *n* being any whole Numbers), the Numerator of the Fraction (whatever it be), drawn into ſome Power of 10, cannot be diviſible by the ſaid Denominator; conſequently the Quotient will never terminate, and therefore muſt neceſſarily be either a repeating, or a circulating Decimal.

Hence it is evident, that in thoſe Decimals which happen to terminate, the Unit's Place of the Diviſor (or Denominator) will ever be found, either 0, 2, 4, 5, 6 or 8; but when either 1, 3, 7 or 9 ſtands in the Unit's Place of the Diviſor (or Denominator) it will be impoſſible for the Quotient (or Decimal Figures) to terminate. *N. B.* If any of the Digits, 2, 4, 5, 6 or 8, ſtands in the Place of Units in the Diviſor, the Quotient or Decimal Figures may, ſometimes, happen not to terminate.

RULE.

Let a competent Number of Cyphers be annexed to the Numerator, to form a Dividend; which being divided by the Denominator, the Quotient (if there happens to be no Remainder) will be *precisely* the Decimal Fraction sought.*

EXAMPLES.

How must $\frac{3}{4}$ and $\frac{7}{8}$ of an Unit, or Integer, be expressed in Decimal Fractions?

OPERATIONS.

4)3.00(.75	8(7.000(.875
28	64
20	60
20	56
. .	40
	40
	. .

Hence

* A Vulgar Fraction cannot *precisely* be expressed in a Decimal Fraction, unless the Denominator of the Vulgar Fraction is either some Power of 2, some Power of 5, or else some Power of 2 into some Power of 5; that is, universally, except the Denominator is $2^m \times 5^n$; supposing m and n to denote any whole Numbers whatever.

For it is very evident, that the Number 10 is divisible by none of the Digits, except 2 and 5; and it is proved (*Eu.* 8. *B.* 14. & 15. *Prop.*) if one Number measure another, that the Square (or Cube) of that Number will measure the Square (or Cube) of the other Number; from whence it follows, that any Power of the greater is divisible by the same Power of the less Number; and it is well known, that, in reducing any Vulgar Fraction into a Decimal Fraction, the Numerator of the former is ever multiplied (or supposed to be multiplied) by some Power of 10, and the Product divided by the

Hence it appears, that $\frac{3}{4}$ (Three-fourths) of an Unit, are equivalent to .75 ($\frac{75}{100}$), that is, to 75 Hundredth Parts of an Unit or Integer: Alſo, $\frac{7}{8}$ (Seven-eighths) of an Unit, are equal to .875 ($\frac{875}{1000}$), *i. e.* to 875 Thouſandth Parts of an Unit.

After the very ſame Manner, may any other Vulgar Fraction be reduced into a Decimal Fraction.

Addition *of* Decimals.

Prop. 2.

To find the Sum of any given Number of Decimal Fractions, or mixed Numbers.

The Method of Operation is the very ſame as in whole Numbers, ſtrict Regard being taken in placing the ſeparating Points one under another; and alſo to place Units under Units, &c. in the Integers or whole Numbers, and Tenths under Tenths, &c. in the Decimal Parts; and laſtly, to point off as many Decimals in the Total, as there are in that given Term which conſiſts of the greateſt Number of Decimal Places.

Example.

Suppoſe it were required to find the Sum of the following mixed Numbers and Decimal Parts; *viz.*

26.489,

---

the Denominator: Hence it is plain, that unleſs the Denominator of the Vulgar Fraction be expreſſed by $2^m \times 5^n$ (*m* and *n* being any whole Numbers), the Numerator of the Fraction (whatever it be), drawn into ſome Power of 10, cannot be diviſible by the ſaid Denominator; conſequently the Quotient will never terminate, and therefore muſt neceſſarily be either a repeating, or a circulating Decimal.

Hence it is evident, that in thoſe Decimals which happen to terminate, the Unit's Place of the Diviſor (or Denominator) will ever be found, either 0, 2, 4, 5, 6 or 8; but when either 1, 3, 7 or 9 ſtands in the Unit's Place of the Diviſor (or Denominator) it will be impoſſible for the Quotient (or Decimal Figures) to terminate. *N. B.* If any of the Digits, 2, 4, 5, 6 or 8, ſtands in the Place of Units in the Diviſor, the Quotient or Decimal Figures may, ſometimes, happen not to terminate.

26.489, 82.05, 18.407, .5632, .82 and .076.— These Terms, being placed according to the preceding Directions, will ſtand thus:

	26.489
	82.05
	18.407
	.5632
	.82
	.076
Total	128.4052

This being ſo very obvious, it would be quite unneceſſary to give any more Examples of this Kind.

SUBTRACTION of DECIMALS.

PROP. 3.

To find the Difference of any two given Decimal Fractions, or mixed Numbers.

RULE.

Let the ſame Method be obſerved in placing the two given Quantities, as in the preceding Rule; and if it ſo happens, that the upper (or greater) *Quantity* ſhould not conſiſt of as many Decimal Places as the *lower*, the Defect muſt be ſupplied by annexing Cyphers (or ſuppoſing them annexed) to the upper Term; then ſubtract as if they were whole Numbers, and we ſhall obtain the Remainder or Difference; obſerving to place the Decimal, or ſeparating Point,

Point, exactly under those of the two given Numbers.

EXAMPLES.

From	8.75	25.87	.76
Take	4.8476	12.384	.2847
Remainders	3.9024	13.486	.4753

MULTIPLICATION *of* DECIMALS.

PROP. 4.

To find the Product of any two given Decimal Fractions, or mixed Numbers.

RULE.

Let that *Factor* which consists of the greatest Number of Figures, be multiplied by the *other*, in the very same Manner as if they were both whole Numbers; and from the Product point off, from the Right-Hand, as many Decimals, as there are Decimal Places in the given Factors.

EXAMPLE I.

Let it be required to find the Product of these two Factors; *viz.* .764 and .28.

```
  .764
   .28
 ------
  6112
 1528
 ------
.21392 the required Product.
```

Note;

Note. The Reaſon of pointing off as many Decimals in the Product as there are Decimal Places in both the given Factors, is very evident: For let the above Fractions be expreſſed by $\frac{764}{1000}$ and $\frac{28}{100}$; then the Product of theſe (by the Nature of Vulgar Fractions) will be $\frac{21392}{100000}$: Moreover, according to the Notation of Decimal Fractions, the *(ſuppoſed)* Denominator of each Fraction muſt conſiſt (as above) of as many Cyphers, as there are Figures (and Cyphers) in both Fractions: Hence it is evident, that the *(ſuppoſed)* Denominator of the Product will conſiſt of an Unit, with as many Cyphers annexed, as there are Cyphers in both the *(ſuppoſed)* Denominators of the given Factors, or Decimal Places in both the given Fractions; and which muſt, evidently, be equal to the Number of Decimals in the required Product.

EXAMPLE 2.

Multiply	24.8763
By	3.47
	1741341
	995052
	746289
Product	86.320761

When the Product does not conſiſt of as many Figures or Places, as there are Decimal Places in both the given Factors, the Defect muſt be ſupplied, by prefixing Cyphers to the Product: As in the following

EXAMPLE;

Example.

Multiply	5.478
By	.00054
	21912
	27390
Product	.00295812

Division *of* Decimals.

Prop. 5.

Two Decimal Fractions, or mixed Numbers (or one of them a mixed Number, and the other either a Decimal Fraction or a whole Number) being given; to find the Quotient arising from dividing one *by the* other. The Method of Operation, here, is the very ſame as in Diviſion of whole Numbers; the only Difficulty lies in determining the true Value of the Quotient, or of pointing off the right Number of Decimal Places. — To effect which, obſerve the following general

Rule.

Point off as many Decimals in the Quotient, as the *Number* of Decimal Places in the Dividend exceeds *that* in the Diviſor.

For it is evident, from the preceding *Note*, that the *Decimal Places* in the Dividend muſt be *exactly* equal to the Number of *thoſe* in both the Diviſor and Quotient.

EXAMPLE I.

Divide .728654 by .34.

OPERATION.

```
.34).728654(2.1431
     68
     --
      48
      34
      --
      146
      136
      ---
       105
       102
       ---
         34
         34
         --
         ..
         --
```

In the above Example, there are ſix Places of Decimals in the Dividend, and only two in the Diviſor; conſequently, by the general Rule, there muſt be four Places of Decimals in the Quotient.

It is very obvious, from the preceding Rule, that when there are juſt as many Decimal Places in the Dividend as there are in the Diviſor, the Quotient will be a whole Number, if there happens to be no Remainder after the Operation: But if there ſhould be a Remainder; let Cyphers be annexed thereto, and ſo continue the Diviſion at Pleaſure, and we ſhall have as many Decimals in the Quotient as there were Cyphers annexed in the Operation.

EXAMPLE

EXAMPLE 2.

Let it be required to divide 1341.482 by 5.283; and to have three Places of Decimals in the Quotient.

It is very evident, from the preceding general Rule, that there must be three Cyphers annexed to the Dividend; then the Operation will be as follows:

OPERATION.

```
5.283)1341.482000(253.924, &c.
      10566
      -----
       28488
       26415
       -----
        20732
        15849
        -----
         48830
         47547
         -----
          12830
          10566
          -----
           22640
           21132
           -----
            1508 Remainder.
            ----
```

When there are not so many Figures in the Quotient, as there are Decimal Places in the Dividend more than in the Divisor; supply the Defect with Cyphers, prefixed to the said Quotient: As in the following

EXAMPLE 3.

Divide 7.856249 by 165.45.

OPERATION.

```
136.45)7.856249(.0575, &c.
        68225
        ------
        103374
         95515
        ------
         78599
         68225
        ------
         10374 Remainder.
```

PROP. 6.

To reduce Coin, Weight, Measure or Time, &c. *into Decimal Fractions.*

RULE.

When there are two, or more, different Denominations given to be reduced into Decimal Fractions, whether they be Coin, Weight, *&c.* First reduce all those different Denominations into the lowest of them, which will be the Numerator of a Vulgar Fraction, whose Denominator will be the given Integer, reduced into the same Denomination as the above-mentioned Numerator; this Vulgar Fraction being then reduced (by *Prop.* 1.) into a Decimal Fraction, will be the Answer sought.

EXAMPLE 1.

Reduce 16*s*. 4*d*. into the Decimal of a Pound.

First,

First, 16*s.* 4*d.* is equal to 196 Pence, the Numerator, and in 20*s.* (the given Integer) are 240 Pence, the Denominator; then (by *Prop.* 1.) $\frac{196}{240}$ being reduced into a Decimal Fraction will be the Answer required.

OPERATION.

240)196.0000(.8166, &c. the Decimal Fraction [sought.
1920
———
400
240
———
1600
1440
———
1600
1440
———
.160 Remainder,

EXAMPLE 2.

Let it be required to express 3*q.* 14*lb.* 10*oz.* in a Decimal Fraction, when one Ton is supposed the Integer, or Unit: Or, which is the same Thing, to find what Decimal Part of a Ton, is 3*q.* 14*lb.* 10*oz.*

First, 3*q.* 14*lb.* 10*oz.* is equal to 1578*oz.* the Numerator:

And one Ton is equal to 35840*oz.* the Denominator:

Then (by *Prop.* 1.) reduce the Vulgar Fraction $\frac{1578}{35840}$, into a Decimal Fraction: See the following

OPERATION.

OPERATION.

35840)1578.000000(.044029, &c. the Decimal sought.

```
35840)1578.000000(.044029, &c.
       143360
       ------
        144400
        143360
        ------
          104000
           71680
          ------
          323200
          322560
          ------
             640 Remainder.
```

PROP. 7.

To find the Value of any given Decimal Fraction.

RULE.

Multiply the given Decimal by that Number (of the next inferior Denomination) which expresses the Value of the Integer, of which the given Decimal Fraction is a Part; and from the Product point off the Decimal Places, according to the Rule observed in Multiplication; and we shall then obtain the Value of the given Decimal in the same Denomination of the Multiplier: And so by proceeding in the same Manner, till we come to the lowest Denomination of the proposed Integer, we shall, at last, get the Value of the proposed Decimal Fraction: The following Examples will render this Rule very plain.

EXAMPLE

EXAMPLE 1.

Required the Value of .9495 of a Pound Sterling.

OPERATION.

$$\begin{array}{rl} .9495 & \\ 20 & \text{Shillings, the Value of the Integer or [Pound.} \\ \hline 18.9900 & \\ 12 & \text{Pence, the Value of the Integer or [Shilling.} \\ \hline 11.88 & \\ 4 & \text{Farthings, the Value of the Integer or [Penny.} \\ \hline 3.52 & \\ \hline \end{array}$$

Hence it appears, that the Value of .9495 of a Pound Sterling, is 18*s.* 11*d.* $\frac{3}{4}$ and 52 Hundredths of a Farthing.

EXAMPLE 2.

How many Quarters, Pounds, Ounces and Drams, are contained in .482 of an Hundred-Weight?

OPERATION.

Operation.

$$
\begin{array}{rl}
.482 & \\
4 & \text{Quarters in the Hundred, or Integer.} \\
\hline
1.928 & \\
28 & \text{Pounds in the Quarter, or Integer.} \\
\hline
7424 & \\
1856 & \\
\hline
25.984 & \\
16 & \text{Ounces in the Pound, or Integer.} \\
\hline
5904 & \\
984 & \\
\hline
15.744 & \\
16 & \text{Drams in the Ounce, or Integer.} \\
\hline
4464 & \\
744 & \\
\hline
11.904 & \\
\hline
\end{array}
$$

Hence it appears, that .482 of an Hundred-Weight, is equal to 1*q.* 25*lb.* 15*oz.* 11*dr.* and 904 Thousandths of a Dram.

Example 3.

To find how many Weeks, Days, Hours, *&c.* are contained in .856 of a Month (*i. e.* four Weeks).

Operation.

OPERATION.

```
   .856
      4 Weeks in the Month, or Integer,
  ------
  3.424
      7 Days in the Week, or Integer,
  ------
  2.968
     24 Hours in the Day, or Integer,
  ------
   3872
  1936
  ------
 23.232
     60 Minutes in the Hour, or Integer,
  ------
 13.920
     60 Seconds in the Minute, or Integer,
  ------
 55.200
  ------
```

It is found, by the preceding Operation, that .856 of a Month, is equivalent to 3 *W.* 2 *D.* 23 *H.* 13 *M.* 55 *S.* and 2 Tenths of a Second.

SECTION II.

Of the SQUARE ROOT.

TO extract the Square Root of any given Number, is to find such a Number (if possible), *which being multiplied by itself; the Product shall be equal to the given Number.* Thus, the Square Root of 4 is 2 (because 2 multiplied by 2 is equal to 4) and for the same Reason, the Square Root of 9 is 3, and of 16 is 4, &c. all which will evident-

ly appear from the following Table of Roots and Squares.

Roots	1	2	3	4	5	6	7	8	9	&c.
Squares	1	4	9	16	25	36	49	64	81	&c.

The Square Roots of Numbers are either ſimple or compound; *viz. ſimple*, when the Root conſiſts of one Figure only; and *compound*, when it contains more than one Figure: And it may be proper to obſerve here, that the Number of Places in the Square of any given Number, whether a ſimple or compound Root, will either conſiſt of juſt double the Number of Places in the ſaid Root, or one Place leſs than the ſaid double; that is, if there are two Places, the Square thereof cannot conſiſt of more than four Places, nor leſs than three; if there are three Places, the Square thereof will either conſiſt of five or ſix Figures, or Places, *&c.* —— Hence it appears (ſee the ſubſequent *Lemma**) that if a *Point* be placed over the Unit's Place of any whole Number, whoſe Square Root is to be extracted, and *another* over the third Figure, and ſo on, over the *fifth*, *ſeventh*, *ninth*, &c. *viz.* over every other Figure to the End; we ſhall have as many integral Figures (or Places) in the Root, as there were Points placed over the propoſed whole Number.

Any whole Number being thus pointed into Periods, its Square Root may be obtained by the following

GENERAL RULE.

First find by the preceding Table, or a few Trials, which of the nine Digits being ſquared, will

will be equal, or the nearest *less*, to the first Period, beginning at the Left-Hand; which being found, place it at the Right-Hand of the given Number, whose Square Root you are then seeking, in the same Manner as a Quotient in common Division. Then let the Square of this Number (which is the first Figure of the required Root) be taken from the first Period, and to the Remainder (if any) join the next Period to the Right-Hand; this Number is called a *Resolvend:* Double the Figure of the Root, and place it as a Divisor to the Resolvend; then seek, as in Division, how often this Divisor is contained in the Resolvend, all but the Unit's Place, and with this Restriction too, that when the Quotient Figure (or this last Figure of the Root) is annexed to the aforesaid Divisor, and the Whole multiplied by the said annexed Figure, the Product shall not exceed the Resolvend, but shall either be equal thereto, or the next less; this Product being taken from the Resolvend, to the Remainder let another Period be annexed, which will then form a second Resolvend.

Double the two Figures of the Root, which place (as before) for a Divisor to this second Resolvend: Find how often this Divisor is contained in the said Resolvend, neglecting the Unit's Place; still observing, that when the Quotient Figure (which is the third Figure of the Root) is annexed to this last Divisor, and the Whole multiplied by the Figure so annexed, the Product must be equal, or the next less, to the Resolvend.

Proceed in this Manner, Period after Period, till they are all brought down; and if there be no Remainder after the Operation, the Number proposed is a square Number.

If there should *still* be a Remainder; then the proposed Number is called a *surd Number*, and has

ly appear from the following Table of Roots and Squares.

Roots	1	2	3	4	5	6	7	8	9	&c.
Squares	1	4	9	16	25	36	49	64	81	&c.

The Square Roots of Numbers are either ſimple or compound; *viz. ſimple*, when the Root conſiſts of one Figure only; and *compound*, when it contains more than one Figure: And it may be proper to obſerve here, that the Number of Places in the Square of any given Number, whether a ſimple or compound Root, will either conſiſt of juſt double the Number of Places in the ſaid Root, or one Place leſs than the ſaid double; that is, if there are two Places, the Square thereof cannot conſiſt of more than four Places, nor leſs than three; if there are three Places, the Square thereof will either conſiſt of five or ſix Figures, or Places, *&c.* —— Hence it appears (ſee the ſubſequent *Lemma**) that if a *Point* be placed over the Unit's Place of any whole Number, whoſe Square Root is to be extracted, and *another* over the third Figure, and ſo on, over the *fifth*, *ſeventh*, *ninth*, &c. *viz.* over every other Figure to the End; we ſhall have as many integral Figures (or Places) in the Root, as there were Points placed over the propoſed whole Number.

Any whole Number being thus pointed into Periods, its Square Root may be obtained by the following

GENERAL RULE.

Firſt find by the preceding Table, or a few Trials, which of the nine Digits being ſquared, will

will be equal, or the nearest *less*, to the first Period, beginning at the Left-Hand; which being found, place it at the Right-Hand of the given Number, whose Square Root you are then seeking, in the same Manner as a Quotient in common Division. Then let the Square of this Number (which is the first Figure of the required Root) be taken from the first Period, and to the Remainder (if any) join the next Period to the Right-Hand; this Number is called a *Resolvend:* Double the Figure of the Root, and place it as a Divisor to the Resolvend; then seek, as in Division, how often this Divisor is contained in the Resolvend, all but the Unit's Place, and with this Restriction too, that when the Quotient Figure (or this last Figure of the Root) is annexed to the aforesaid Divisor, and the Whole multiplied by the said annexed Figure, the Product shall not exceed the Resolvend, but shall either be equal thereto, or the next less; this Product being taken from the Resolvend, to the Remainder let another Period be annexed, which will then form a second Resolvend.

Double the two Figures of the Root, which place (as before) for a Divisor to this second Resolvend: Find how often this Divisor is contained in the said Resolvend, neglecting the Unit's Place; still observing, that when the Quotient Figure (which is the third Figure of the Root) is annexed to this last Divisor, and the Whole multiplied by the Figure so annexed, the Product must be equal, or the next less, to the Resolvend.

Proceed in this Manner, Period after Period, till they are all brought down; and if there be no Remainder after the Operation, the Number proposed is a square Number.

If there should *still* be a Remainder; then the proposed Number is called a *surd Number*, and has

no *true* Root; but any Degree of Exactneſs may be obtained, by annexing two Cyphers to each Remainder, and proceeding as above.

EXAMPLE 1.

Let it be required to extract the Square Root of 134689: Or, which is the ſame Thing, to find a Number (if poſſible), which being multiplied by itſelf, the Product ſhall be equal to 134689.

OPERATION.

The given Number being pointed in the Manner as before taught, will ſtand thus, $1\dot{3}4\dot{6}8\dot{9}$; which ſhews there will be three Figures in the Root, if it happens to be a perfect ſquare Number; and if a ſurd Number, there will, however, be three Figures in the integral Part of the Root.

Here then being three Periods; *viz.* 13, 46 and 89 (or more properly 130000, 4600 and 89): Firſt find in the Table of ſimple Roots, or otherwiſe, what Number being ſquared, will be equal, or the next leſs, to the firſt Period 13, which is readily found to be 3, the firſt Figure of the Root; the Square of which (9) being taken from 13, leaves 4; to this Remainder join the next Period, and it makes 446, which is called the *Reſolvend*; and the Work will ſtand as under.

$$
\begin{array}{l}
1\dot{3}4\dot{6}8\dot{9}(3 \\
9 \\
\hline
446 \text{ Reſolvend.}
\end{array}
$$

Then

Then place (6) the Double of the Root, as a Diviſor to this Reſolvend 446; and ſeek how often 6 in 44; but in ſuch a Manner, that the Quotient Figure (which will be the ſecond Figure of the Root) being annexed to the Diviſor (6) and the Whole multiplied by the Figure ſo annexed, the Product muſt be either equal, or the next leſs, to the Reſolvend 446: Now, in the preſent Example, the ſecond Figure of the Root is found to be 6, and therefore the Diviſor is 66, which being multiplied by 6, and the Product (396) taken from the Reſolvend (446) leaves 50; to which join the next Period (89) and it makes 5089, for a ſecond Reſolvend; and the Operation will ſtand as follows.

$$
\begin{array}{rl}
 & 1\dot{3}4\dot{6}8\dot{9}(36 \\
 & 9 \\
\hline
66) & 446 \\
 & 396 \\
\hline
 & 5089 \textit{ Reſolvend.}
\end{array}
$$

Double the Figures (36) of the Root, which place as a Diviſor to this laſt Reſolvend; then find how often 72 (the Double of 36) is contained in 508, and with the ſame Reſtriction as before; namely, when the new Quotient Figure (which will now be the third Figure of the Root) is annexed to (72) the Diviſor, and the Whole multiplied by the Figure ſo annexed, the Product ſhall not exceed the Reſolvend; but ſhall be either equal thereto, or the next leſs: Now here it is eaſy to perceive that 7 is the next Figure of the Root (for 7 Times 72 is 504); therefore annex 7 to the Diviſor, and multiply the Whole (727) by 7, the Product will be

be 5089, which being equal to the Resolvend, (and all the Periods brought down) shews the proposed Number (134689) is a perfect square Number: See the whole Operation.

```
 . . .
134689(367 the required Root.
  9
 ------
 66)446
    396
 ------
727)5089
    5089
 ------
    ....
 ------
```

When the given Number to be extracted, is either a mixed Number, or a Decimal Fraction, the Method of Operation will be the very same as in the foregoing Example; only observing, that if the Decimals consist of an odd Number of Places, they must first be made an even Number, by annexing 1, 3, 5 or 7, *&c.* Cyphers according to the Exactness required in the Root; which Root will always consist of as many Integral, and as many Decimal Places, as there were Points respectively placed over the Integers, and Decimal Places (together with the Cyphers annexed) of the proposed Number.

EXAMPLE

LEMMA.

* *If any Number whatever be denoted by F Places; then will the Square of that Number ever consist of either 2F, or 2F—1 Places.*

It is sufficiently evident, that if any Number of Figures, denoted by F, be multiplied by any one of the nine Digits, the Number of Places in the Product cannot be less than F (the Number of Figures to be multiplied), nor greater than F+1; the Number of Places which would arise by multiplying F Number of Places by 10: And, it is equally plain, if the Multiplier consists

Example 2.

What is the Square Root of 184.2?

First, let three Cyphers be annexed and pointed as before directed, and the Operation will be as follows.

```
 184.2000(13.57, &c.
 1
 ----
 23)84
    69
 -----
 265)1520
     1325
 --------
 2707)19500
      18949
 ----------
        551 Remainder.
```

Here

fists of two Places, the Number of Places in the Product cannot be more than F+2; *viz.* the Number of Places which would be produced, by multiplying F Places by 100; nor less than F+1; the Number of Places which would arise from F Places being multiplied by 10; which is the least Multiplier for any two *integral* Figures.

By the very same Method of Reasoning it appears, that, if the Multiplier consists of three Places, the Number of Places in the Product cannot be greater than F+3; the Number of Places (*i. e.* Figures and Cyphers) produced, by multiplying F Places by 1000; nor can it be less than F+2; the Number of Places which would arise by multiplying F Places by 100: Hence the Number of Places, in the three aforesaid Cases, may be either F or F+1, F+1 or F+2, F+2 or F+3, according to the Largeness, or Smallness, of the last Figure on the Left-Hand, in the Multiplicand and the Multiplier.

Now let the Number of Places in the greater Factor (or Multiplicand) be here denoted by F, and those in the less Factor (or Multiplier) by f: Then it is evident, from the preceding Method of Reasoning, that the Number of Places in the Product will be either $F+f$, or $F+f-1$; consequently when the Number of Places in each Factor is equal; that is, when $F=f$ (which is the Case when any Number is to be squared); then the

Here must be two Decimals pointed off in the Root (because there are two Points over the Decimal Places) and also two integral Numbers, agreeable to the foregoing Observation.

By annexing more Cyphers, and continuing the Operation, we may approximate the Value of the Square Root of 184.2, to any assigned Degree of Exactness.

EXAMPLE 3.

To extract the Square Root of .84567.

OPERATION.

.845670(.919 the required Root, *nearly*.
81

181)356
181

1829)17570
16461

1109

EXAMPLE

---

the Number of Places in the Product, or in the Square of F (f) Number of Places, will be either 2F, or 2F—1; that is, if F (or f) = 1 (one Figure), the Number of Places in F^2 (f^2) will be either 1 or 2 (*viz.* 2F—1 or 2F); if F=2 (two Places), the Number of Places in F^2 will be either 3 or 4 (*viz.* 2F—1, or 2F), and if F=3 (three Places); then will the Number of Places in F^2, be either 5 or 6 (*viz.* 2F—1, or 2F), *&c.* Q. E. I.

COROLLARY I.

Hence it appears, that if any square Number consists of either 1 or 2 Figures or Places, its Square Root will consist of one Figure *only*; if there be either 3 or 4 Places in any square Number, its Square Root will have *precisely* two Places; if either 5 or 6, its Square Root will have three Places, *&c.*

EXAMPLE 4.

What is the Square Root of 2?

OPERATION.

```
2.00000000(1.4142, the Square Root
1                          [of 2, nearly.
—
24)100
    96
   ———
 281)400
     281
     ———
 2824)11900
      11296
      —————
 28282)60400
       56564
       —————
        3836
        ————
```

E SECTION

&c. From whence appears the Reason of pointing the 1st. 3d, 5th, and 7th Place, &c. beginning at the Unit's Place of any Number, whose Square Root is to be extracted.

COROLLARY 2.

It appears likewise, from the foregoing *Lemma*, if any Number of Figures be represented by F, that the Number of Places in F^3 cannot exceed 3F, nor be less than 3F — 2; therefore the Number of Figures in the Cube of one single Figure, will be either 1, 2 or 3 (*viz.* 3F — 2, 3F — 1 or 3F); the Number of Places in the Cube of any two Figures, will be either 4, 5 or 6 (*viz.* 3F — 2, 3F — 1 or 3F); and the Cube of three Figures, will consist of either 7, 8 or 9 Places (*viz.* 3F — 2, 3F — 1 or 3F): —Hence the Reason is plain for pointing the 1st, 4th, 7th, 10th Places, &c. beginning at the Unit's Place of any whole Number, whose Cube Root is to be extracted.

SECTION III.

Of the CUBE ROOT.

TO extract the Cube Root of any given Number, is the same Thing as to find (if possible) *a Number, which being multiplied by itself, and the Product thereof multiplied again by the said Number; the last Product shall be equal to the Number given.*—— For Instance, the Cube Root of 27 is evidently 3; because 3 multiplied by 3, and the Product (9) multiplied again by 3 gives 27: Also the Cube Root of 64 is 4, of 125 is 5, *&c.* see the following Table.

Roots	1	2	3	4	5	6	7	8	9	&c.
Cubes	1	8	27	64	125	216	343	512	729	&c.

The *Cube Roots* of Numbers are simple, when they consist of one Figure only; and compound, when they contain more than one: The first of these are easily learnt by Heart, from the preceding Table; but the latter require a tedious Operation: To effect which, observe the following Directions.

Make a Point over the Unit's Place of the proposed Number, and another over the 4th, and so on, over the 7th, 10th Figure, *&c.* and we shall have as many integral Figures in the Root, as there are Points placed over the given whole Number.

Find a Number, which, being cubed, shall be equal, or the next less, to the first Period, beginning at the Left-Hand; this Number is the first Figure of the Root.

The

The Cube of this first Figure of the Root being taken from the first Period, to the Remainder (if any) bring down the next Period, which will then form what is called the *Resolvend.*

Triple the Root, and also triple its Square, which put down, so that the Unit's Place of the latter may stand under the Place of Tens of the former; the Sum of these Numbers is called a *Divisor*, by which the next Figure of the Root may be nearly estimated, as follows.

Seek how often the Divisor is contained in the Resolvend, exclusive of the Unit's Place thereof, and with the following Restriction too; namely, that if the Cube of the Quotient Figure (which must be the second Figure of the Root) be placed under the Resolvend, Units under Units, and the Square of the Quotient Figure multiplied by the triple of the other Figure of the Root, and the Unit's Place of the Product, set under the Place of Tens of the aforesaid Cube, and also this last Quotient Figure (or second Figure of the Root) multiplied by the triple Square of the first Figure of the Root (found as above) and the Unit's Place of this Product set under the Place of Tens of the last Product; the Sum of these three Numbers (which is called the *Subtrahend)* must be equal, or the next less, to the Resolvend; from which let the Subtrahend be taken, and to the Remainder (if any) bring down the next Period to form another Resolvend: Proceed in the very same Manner, as above, to find the Divisor and the third Figure of the Root, and so on, Period after Period, till they are all brought down; then if there happens to be no Remainder, the Number proposed was a perfect cube Number: But if the whole Number to be extracted, be not a perfect cube Number, three Cyphers must be annexed

to the laſt Remainder for a new Reſolvend, and ſo proceed as above; then as many Times as there are three Decimal Cyphers annexed to the Remainders, ſo many Decimal Places will be in the Root.

EXAMPLE I.

To extract the Cube Root of 3375.

OPERATION.

3375(15 the required Root.
1

2375 Reſolvend.

3 Triple of the Root 1.
3 Triple Square of the Root.

33 Diviſor.

125 The Cube of 5.
75 The Square of 5, by the triple Root.
15 Triple Square of the Root by 5.

2375 Subtrahend, to be taken from the Reſolvend.

0 Remains.

Otherwiſe, more generally; *from whence will appear the Reaſon of placing the Numbers to form the Diviſor and Subtrahend, as above.*

It is plain, if the above given Number (3375) be pointed according to the foregoing Directions, there will be two Periods, *viz.* 3000 and 375, which ſhew there will be two Figures in the Root. The

The next less Cube Number to 3000 is 1000, whereof the Cube Root is 10; therefore 1000 (the Cube of the Root 10) being taken from the first Period, there remains 2000; to which add the next Period (375), and we get 2375 for a Resolvend: See the following Operation at large.

3375(15 the required Root.
1000

2375 Resolvend.

---

30 Triple of the Root 10.
300 Triple Square of the Root 10.

---

330 Divisor.

---

125 The Cube of 5.
750 The Square of 5, by the triple Root 10.
1500 Triple Square of the Root (10) by 5.

2375 Subtrahend, to be taken from the Resolvend.

0

EXAMPLE 2.

To extract the Cube Root of 22425768.

OPE-

OPERATION.

```
22425768(282
 8
-----
14425     Resolvend.
-----
    6     Triple of the Root.
   12     Triple Square of the Root.
-----
  126     Divisor.
-----
  512     Cube of 8.
 384      Square of 8, by the triple Root.
 96       Triple Square of the Root by 8.
-----
13952     Subtrahend, to be taken from the above
                                        [Resolvend.
  473768  Resolvend.
  -------
      84  Triple of the Root
    2352  Triple Square of the Root.
  -------
   23604  Divisor.
  -------
       8  Cube of 2.
     336  Square of 2, by the triple Root.
   4704   Triple Square of the Root by 2.
  -------
  473768  Subtrahend.
  -------
       0  Remains.
  -------
```

SECTION

SECTION IV.

THE CONSTRUCTION AND USE OF THE SLIDING-RULE.

TO treat of this valuable Inſtrument from its Origin, it would be abſolutely neceſſary to explain the Nature and Properties, and to compute a Table, of Logarithms; from whence the principal Lines thereon were conſtructed: But, as theſe Things might be deemed foreign to the preſent Subject, I ſhall therefore content myſelf with giving the Method of conſtructing the Lines, and afterwards of applying them to Practice: However, for the Sake of the inquiſitive Reader, I ſhall ſhew, in the *Notes* ſubjoined, the Conformity of the Operations on the Rule with the Nature of Logarithms.

The Lines on this Inſtrument, marked A, B, N, C, and two marked D, are Lines of Numbers, commonly called *Gunter's Lines*, from their worthy Inventor Mr. *Edmund Gunter*, the third Profeſſor of Aſtronomy in *Greſham College*, LONDON; who, in the Year 1624, firſt made the Diſcovery of applying Logarithms to Extenſion; and of performing, with great Facility, by Means of a Pair of Compaſſes, and the ſaid Line of Numbers, the Buſineſs of Multiplication, Diviſion, and all Arithmetical Operations, where the Rule of Proportion was required: But the Uſe of Compaſſes being found both troubleſome and liable to Error, the late ingenious *Tho. Everard*, Eſq. made a very conſiderable Improvement in the Application of the Line of Numbers, by contriving one Line to ſlide

ſlide by another, in the ſame Manner as the Inſtrument we are now ſpeaking of.

The Method of conſtructing the Line of Numbers is the very ſame, let the Radius, or Length of the Line, be what it will; thoſe *Lines* on the Sliding-Rule, marked A, B, N, and C, are graduated upon Half the Radius of that *Line* which is marked D.

Let a Line, or Rule (equal to the whole Diſtance or the intended Radius), upon which the Line of Numbers is to be graduated, be divided into 1000 equal Parts; then with your Compaſſes take, from this Line of equal Parts, the Numbers expreſſing the Logarithms (to the firſt three Places of Decimals, omitting the Characteriſtics) of 101, 102, 103, 104, &c. progreſſively to 1000, and apply them ſucceſſively from 1 (the Beginning of the Radius), and we ſhall thereby mark out all the Diviſions on the ſingle Radius D: But the moſt expeditious and exact Method of forming a Line of Numbers, is as follows.

Open the Sector till the Diſtance of the two Braſs Pins on the Line of Lines (marked L. L.) be equal to the Length of the intended Radius: Place 1 (the Logarithm of which is 0) at the Beginning of the Line, towards the Left-Hand; then, according as the Space between 1 and 2 is divided into 100 or 50 Parts (as in the ſingle Radius marked D, or thoſe Radii marked A, B. &c.), take from the Sector, opened to the intended Radius, the Diſtances, or Numbers, anſwering to (at leaſt the three firſt Places of Decimals) the Logarithms of 1.01, 1.02, 1.03, 1.04, &c. to 2 (if the ſingle Radius D); or thoſe of 1.02, 1.04, 1.06, 1.08, &c. to 2 (if any of the Radii marked A, B, &c.); then theſe Diſtances being ſucceſſively applied

plied from 1, along the Line D or A, will mark out all the Divisions between 1 and 2 on those Lines respectively.

Now the Distance of the Divisions 2 and 3, in the single Radius (marked D), is divided into 50 Parts; but on those Radii marked A, B, &c. the said Distance is divided into 20 Parts; therefore, for the former, take from the Sector, opened for the single Radius, the Logarithms of 2.02, 2.04, 2.06, 2.08, &c. to 3; and for the latter, take from the Sector, opened for the double Radius, the Logarithms of 2.05. 2.1, 2.15. 2.2, 2.25, &c. to 3, and apply these Distances respectively from 1, along the Lines D and A, and we shall thereby obtain all the Divisions between 2 and 3: Moreover, the Distance between 3 and 4, in each Radius on the Rule, is divided into 20 Parts; therefore take from the Sector (opened to its proper Radius) the Logarithms of the Numbers 3.05, 3.1, 3.15, 3.2, 3.25. &c. to 4, and apply them (as before) from 1, along either of the Lines D, A, or B, and they will point out all the Divisions between 3 and 4: By proceeding in this Manner the Divisions between 4, 5; 5, 6; 6, 7; 7, 8; 8, 9; and 9 and 10, may be easily marked out.

The Line marked E, consisting of three equal Radii, is constructed in the very same Manner as the above: For the Sector being opened to one-third of the Extent of the single Radius D; take off the Logarithms of 1.05, 1.1, 1.15, &c. to 2, and apply those Distances from 1, 10, or 100, in each Radius, and they will mark out all the Divisions between 1 and 2, 10 and 2, and 100 and 2: — Again, the Distance between 2 and 3 is divided into 20 Parts; proceed therefore according to the foregoing Directions, and we shall obtain all the Divisions between those two Numbers;

and in like Manner between 3, 4; 4, 5; and 5 and 6, *&c.*

On the Line marked M.D, are also placed Lines of Numbers, only they stand in an inverted Order, beginning at 21.5042; the last Division on the Right-Hand will then represent .215042: The Use of this Line (MD), together with A and B (or N), is chiefly confined to the Gauging of Malt, in Vessels in the Form of rectangular Parallelopipedons, at one Setting of the Rule; but the same Examples may be performed, with more Ease to a Learner, by the Lines A and B only; or with the Lines D and C, after finding a Mean-Proportional between the Length and Breadth of the Base.

The Lines of Segments, marked S.S. and S.L. (signifying *Segment Standing* and *Segment Lying)*, are used in finding the Ullage of a Cask, or the Quantity of Liquor which a Cask wants of being full, or what Quantity is in it, if not quite full.

These Lines of Segments may be laid down in the following Manner. Take a Cask whose Content is 100 Ale (or Wine) Gallons, and the nearest in Form to those which most frequently occur in Practice; then suppose the Bung-Diameter or Length (according to the Position of the Cask) divided into 100 equal Parts, which must be laid down on the Slide marked N, in a logarithmic Manner, by the Method already prescribed, *Page* 32.

Draw out, and carefully measure, successively, the Quantities contained in the 1st, 2d, 3d, 4th, and 5th, *&c.* of those equal divisions of the Bung-Diameter (or Length); then place the Quantity contained in the 1st, in the 2 first, 3 first, 4 first, *&c.* of those equal Divisions *exactly* against the Numbers 1, 2, 3, 4, *&c.* respectively, on the

Slide

Slide N; and so by proceeding in this Manner, we shall obtain the true Quantity in such Cask to every hundredth Part of its Bung-Diameter, or Length.—And if either of these were supposed to be divided into any other Number of equal Parts (besides 100), and the Quantities contained in the first, 2 first, 3 first, &c. of those equal Parts, be placed exactly opposite the Numbers, 1, 2, 3, &c. respectively, on the Slide N; we should thereby obtain a Table of Segments for a standing (or lying) Cask similar to the former.

Whence the Reason of finding what is called the *Segment*, is very evident: For it is only conceiving the Bung-Diameter (or Length) of that Cask, from which the Lines of Segments were supposed to be constructed, to be divided into as many equal Parts as there are Inches, &c. in the Bung-Diameter (or Length) of the Cask, whose Ullage we are then seeking, and placing that Number against 100 on the Segments; then opposite any proposed Number of wet Inches, &c. (or equal Parts of the Bung-Diameter or Length) we shall have the Segment sought.

On various Parts of the Rule are several remarkable Points; some of which are distinguished with Brass Pins and Letters, others with only small Dots and Letters.

Thus on the Line A there is marked MB, with a Brass Pin at 2150.42, the cubic Inches in a Malt Bushel; also on the same Line is fixed a Brass Pin, with the Letter A at 282, the cubic Inches in the Ale Gallon.

On the Line B is a small Dot marked at .707, and also the Letters *S. i.* which signify *Square inscribed*; useful in finding the Side of a Square inscribed in any given Circle: At .886 is a small Dot and likewise the Letters *S. e.* which denote

Square equal; uſeful in finding the Side of a Square whoſe Area ſhall be equal to that of any given Circle: At 3.1416 is a ſmall Dot marked C, ſignifying *Circumference*, neceſſary for finding the Circumference of a Circle to any given Diameter.

On the Line D are placed ſeveral Gauge-points, diſtinguiſhed by Braſs Pins and Letters: *Viz.* W.G. with a Braſs Pin, is placed at 17.15, being the Wine Gauge-point for Circles; and A.G. marked at 18.95, ſignifying the Ale Gauge-point for Circles, *&c.* At 46.37 are the Letters M.S. which ſignify *Malt Square*, being the Malt Gauge-point for Square Meaſure: At 52.32 ſtand M.R. which denote *Malt Round*, being the Malt Gauge-point for circular Meaſure: Alſo, at 6.23 ſtand T.P, which ſignify *Tallow Pounds*, being the neat Tallow Gauge-point for circular Figures.

On the Slide C there is a ſmall Dot, with the Letters O.C. marked at .07957, which is the Area of a Circle whoſe Circumference is Unity; uſeful in finding the Area in Inches, Feet, *&c.* of any Circle whoſe Circumference is known: On the ſame Line is marked O.*d.* at .7854, the Area of a Circle whoſe Diameter is Unity; this is uſeful in finding the Area in Inches, Feet, *&c.* of any Circle whoſe Diameter is given.

The Method of eſtimating the Values of the Diviſions on the Sliding-Rule; *and the Uſe thereof.*

WHatever Value is aſſigned to the firſt 1, towards the Left-Hand, (whether 1, 10, 100, *&c.*) on the Lines marked A, B, N, *&c.* the following integral Numbers, 2, 3, 4, *&c.* will repreſent twice, thrice, four times, *&c.* as much;

much; and consequently the second 1 (if a double Radius) will be 10 times the Value of the first; and the third 1 (if a triple Radius) will be 100 times the Value of the first, or 10 times the Value of the second 1. The Values of the integral Divisions being thus estimated, those of the intermediate Divisions may be easily known; being always the Quotient expressed by the Value of the Difference of two adjoining integral Numbers, divided by the Number of Parts contained between them.

Thus, for Example, if the first 1, at the End of the Line A (B or N), stands for *one*, the following 2 for *two*, &c. then the Number of Divisions between 1 and 2 being 50, and the Value of the Difference of those integral Divisions 1; therefore the Value of one of the intermediate Divisions is $\frac{1}{50}$th; consequently the Values of the 1st, 2d, 3d, 4th, 5th, &c. Divisions from 1, will be expressed by $1\frac{1}{50}$, $1\frac{2}{50}$, $1\frac{3}{50}$, $1\frac{4}{50}$, $1\frac{5}{50}$, &c.

Again, the Space between the second 1 and 2 (which Numbers, according to the last Estimation, represent 10 and 20) is also divided into 50 Parts; that is, first into 10 large Divisions, and then each of those into 5 Parts; then the Difference of the integral Numbers (10 and 20) being 10; therefore the Value of *one* large Division will be 1 or $\frac{10}{10}$, (*i. e.* 10 divided by 10), and the Value of one small Division is $\frac{10}{50}$ or $\frac{1}{5}$; consequently the Values of the 1st, 2d, 3d, 4th, 5th, &c. Divisions from 10, will (in this Case) be expressed by $10\frac{1}{5}$, $10\frac{2}{5}$, $10\frac{3}{5}$, $10\frac{4}{5}$, 11, &c. Moreover, if the said integral Numbers 1 and 2, denote 100 and 200; then the Value of one of the (ten) large Divisions will be expressed by 10, or $\frac{100}{10}$; and the Value of one of the (fifty) small Divisions will be expressed by 2, or $\frac{100}{50}$; therefore the Values of the 1st, 2d, 3d,

4th,

4th, *&c.* Divisions from 100, will be represented by 102, 104, 106, 108, *&c.* — By the very same Method of proceeding, the Values of the intermediate Divisions, between any two adjoining integral Numbers, may be known.

Multiplication by the Lines A *and* B, *on the* SLIDING-RULE.

PROP. I.

To find the Product of two given Numbers, by the Sliding-Rule.

RULE.

To either of the given Numbers (or Factors) on A, set 1 on B; then against the other on B, is the required product on A.

EXAMPLE I.

Required the Product of 3 by 8, by the Sliding-Rule.

Set 1 on B, to 3 (or 8) on the Line A; then against 8 (or 3) on B, is 24 on A.*

It may be proper to observe, that it will frequently happen, when 1 on B is set to either of the given Factors on A, the other cannot (according to the *true* Numeration of the Rule) be expressed on the Line B; or, being found thereon, it may perhaps fall

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* By drawing out the Slide, till 1 on B is opposite to 3 on A, it is evident we thence obtain the Sum of the Distances 1 to 3 on A, and 1 to 8 on B: But these Distances are respectively as the Logarithms of 3 and 8; and it is well known that the Sum of the Logarithms of two Numbers will express the Logarithm of their Product; ∵ the Sum of the Distances, 1 to 3 on A, and 1 to 8 on B, will be as the Log. 3 + Log. 8 (=Log. 3 × 8) = Log. 24.

fall beyond the Line A; in such Circumstances it will be most convenient, after setting Unity on B to one of the given Factors on A, to divide the other by some Power of 10 † (*viz.* 10, 100, 1000, *&c.* till the Quotient can be found opposite some Division (or Product) on A; then that Product, thus arising, must be multiplied by the very same Power of 10 as the given Factor was divided by. One Example will make this Observation sufficiently plain.

EXAMPLE 2.

To find, by the Sliding-Rule, the Product of 120 by 95.

First, if 1 on B is set to 120 on A, then will the other Factor (95) fall beyond the Line A:—Again, if 1 on B is set to 95 on A, then the other Factor (120) cannot be found on B; because the greatest Number (in a double Radius) cannot exceed 100, when the first Radius begins with Unity.

But by setting 1 on B to either of the given Factors on A, and looking opposite $\frac{1}{10}$th of the other Factor (which in this Case is sufficient), we shall have $\frac{1}{10}$th Part of the Product sought. Thus, set 1 on B to 120 on A; then against 9.5 on B, is 1140 on A; which, being multiplied by 10, gives 11400, the required Product.

It

† If the given Factors be called m and n, the Product of them r, and the Index of any Power of 10 be denoted by a: Then we shall have, $1 : m :: n : r$ (or $1 : n :: m : r$); $\because 1 : m :: \frac{n}{\overline{10}|^a} : \frac{r}{\overline{10}|^a}$, (or $1 : n :: \frac{m}{\overline{10}|^a} : \frac{r}{\overline{10}|^a}$); hence $\frac{r}{\overline{10}|^a} = \frac{m \times n}{\overline{10}|^a}$; consequently, $\frac{r}{\overline{10}|^a} \times \overline{10}|^a$ $\left(= \frac{m \times n \times \overline{10}|^a}{\overline{10}|^a} = m \times n\right) = r$. Q.E.I.

It may be proper to obſerve, that whether one of the given Numbers be ſet to Unity on the Line A or the Line B, the other given Number (or Factor) muſt be found on the ſame Line where 1 (or Unity) was taken.

Diviſion by the Lines A *and* B, *on the* SLIDING-RULE.

PROP. 2.

To find the Quotient of two given Numbers, by the Sliding-Rule.

RULE.

To the Diviſor on A, ſet 1 on B; then againſt the Dividend on A, is the Quotient on B.

EXAMPLE I.

Let the Dividend be 75, and the Diviſor 5; required the Quotient.

Set 1 on B, to 5 on A; then againſt 75 on A, is 15 on the Line B, the Quotient ſought.‡

It will ſometimes happen, that, when 1 on B is ſet to the Diviſor on A, the Dividend cannot (according to the *true* Numeration of the Rule) be found

‡ When the Slide is drawn out till 1 on B is oppoſite 5 (the Diviſor) on A, we ſhall then get the Difference of the Diſtances of 1 to 5 on A, and alſo 1 to 75 on the Line A, but expreſſed on the Line B from 1 to 15. Now theſe Diſtances are reſpectively as the Logarithms of 5, 75, and 15, and the Difference of the Logarithms of two Numbers being equal to the Logarithm of their Quotient; ∵ the Difference of the Diſtances of 1 to 75 and 1 to 5 on A (= Log. 75 — Log. 5), will be as the Log. of 15 on B.

found on the Line A; therefore, in this Caſe ‖, it will be neceſſary to divide the given Dividend by ſuch a Power of 10, as will bring the Quotient thereof upon the Line A; then againſt this Quotient (*viz.* of the Dividend divided by 10, 100, 1000, *&c.*) is a Number on B, which being multiplied by the ſame Power of 10 as the given Dividend was divided by, we ſhall then obtain the true Quotient ſought.

EXAMPLE 2.

What is the Quotient of 385 divided by 7?

To 7 on A, ſet 1 on B; then as 385 cannot be expreſſed on A, becauſe the ſecond Radius, in this Caſe, ends with 100; therefore let the Number be divided by 10, then oppoſite 3.85 (the Quotient) on A, is 5.5 on B; which being multiplied by 10 gives 55, the Quotient ſought.

It is to be obſerved here, that (whether the Diviſor on A is ſet to 1 on B, or the Diviſor on B is ſet to 1 on A) the Quotient muſt always be found on the ſame *Line* where 1 was taken, and the Diviſor and Dividend on the *other*.

One Example in the *Rule of Three* will be ſufficient, ſince the Method of Operation by the Sliding-Rule is, very nearly, the ſame as in Multiplication; the only Difference is, that inſtead of ſetting 1 on B, to one of the given Factors on A, we muſt ſet the firſt of the three given Terms on

 B,

‖ Let m denote the Diviſor, n the Dividend, and let the Quotient thereof $= r$; alſo let the Index of any Power of 10 be denoted by a; Then $m : 1 :: n : r$, or $m : 1 :: \frac{n}{\overline{10}|^{a}} : \frac{r}{\overline{10}|^{a}}$; $\because \frac{n}{\overline{10}|^{a}} = \frac{mr}{\overline{10}|^{a}}$, or $\frac{n}{m} = r$. Q. E. I.

B, to either the 2d or 3d given Number (or Term) on A; then againſt the other Number on B, we ſhall have the 4th Number, or anſwer ſought, on A.

Example 2.

If 4 Yards of Cloth coſt 14 Shillings, what will 28 Yards coſt, at the ſame Rate?

Set 4 on B, to 14 on A; then oppoſite 28 on B, is 98 on A, the Anſwer ſought §. — Or, ſet 4 on B, to 28 on A; then againſt 14 on B, is 98 on A, (or 4*l.* 18*s.*) the ſame as before.

If it ſhould happen, when the firſt Term (or Number) on B, is ſet to the ſecond or third Number on A, that the other Number on B falls beyond the Stock, or the Line A; then, in ſuch Circumſtance, let that Number, which ſo falls off the Rule, be multiplied or divided (according as it falls off towards the Left or Right-Hand) by ſome Power of 10; and againſt the Product, or Quotient, on B, is a fourth Number on A, which being divided or multiplied by the ſame Power of 10 as the forementioned Number was multiplied or divided by; the Quotient, or Product, will be the 4th Number, or Anſwer ſought.†

Note.

§ By drawing out the Slide till 4 on B ſtands oppoſite 14 on A, we thence obtain, on the Line A, the Diſtance from 1 to 14, *plus* the Diſtance from 1 to 28, *minus* the Diſtance of 1 and 4 (on B); but theſe Diſtances are reſpectively as the Logarithms of 14, 28, and 4; ∴ the Log. 14 + Log. 28 — Log. 4 = Log. $\frac{28 \times 14}{4}$ = Log. 98 on A.

† Let four proportional Numbers be repreſented by m, n, s, and r; alſo let the Index of any Power of 10 be denoted by a: Then we have $m : n :: s : r$, (or $m : s :: n : r$); ∴ $m : n :: \frac{s}{10^a} : \frac{r}{10^a}$, (or $m : s :: \frac{n}{10^a} : \frac{r}{10^a}$); ∴ $\frac{sn}{10^a} = \frac{mr}{10^a}$, or $sn = mr$.

Moreover,

Note. It makes no Difference whether the firſt Number be taken on the Line A or B; only obſerve, that the 4th Number, or Anſwer, muſt be found on the contrary Line to that, whereon the firſt Number was taken. — But, in finding the Areas of plane Figures in Ale Gallons, Malt Buſhels, &c. (as will be ſhewn farther on), it will be found moſt convenient to take the firſt Number *viz.* 282, 2150, on the Line A, as there are generally braſs Pins fixed at thoſe Numbers.

To extract the Square Root, by the SLIDING-RULE.

Set 10 on C to 10 on (the Stock) D; then againſt any propoſed Number on C, is its Square Root on D.

It will be proper to obſerve here, that if the Number, whoſe Square Root is required, conſiſts of an odd Number of integral Places, its Square Root will be found oppoſite the firſt Radius on the Line C: But if there be an even Number of Places, in the Number whoſe Square Root is ſought, then will that Root fall againſt the ſecond Radius on the Line C‡.

 EXAMPLE.

Moreover, $m : n :: s \times \overline{10}|^{a} : r \times \overline{10}|^{a}$; $\therefore n \times s \times \overline{10}|^{a} = m \times r \times \overline{10}|^{a}$; conſequently $ns = mr$ Q. E. I.

‡ The 1 at the End of the Line D, may denote either 1, 10, 100, 1000, &c. therefore the firſt 1 on C oppoſite thereto, muſt, by the Conſtruction of the Lines, ſignify either 1 (1^2), 100 ($\overline{10}|^{2}$), 10000 ($\overline{100}|^{2}$), &c. and conſequently the ſecond 1 on C, will repreſent either 10, 100, 1000, &c.

EXAMPLE.

What is the Square Root of 15376?

Let the Rule be ſet as above directed; then it is evident the firſt 1 on C will repreſent 10000, and the 1 on D (oppoſite thereto) is its Root, which now repreſents 100; likewiſe 5000 will be repreſented by 5 of the large Diviſions on C, and 376 will, very nearly, be repreſented by 2 of the ſmall Diviſions; then againſt this Point on C, we have 124 on the Line D; the Root ſought.

To extract the Cube Root, by the SLIDING-RULE.

Set 10 on (the Slide) D, to 1000 on E; then againſt any propoſed Number on E, is its Cube Root on D.

It will alſo be proper to obſerve here, that if the Number, whoſe Cube Root is ſought, conſiſts of either 1, 4, 7, 10, *&c.* integral Places, its Cube Root will be obtained oppoſite the firſt Radius on E; and if the Number contains either 2, 5, 8, 11, *&c.* Places, its Cube Root will be found oppoſite the ſecond Radius on E; but the Cube Root of a Number, conſiſting of either 3, 6, 9, 12, *&c.* Places, will be had oppoſite the third Radius on E. †

EXAMPLE.

† The 1 on (the Slide) D, may repreſent either 1, 10, 100, 1000, *&c.* and therefore the firſt 1 on E, will repreſent either 1 (1^3), 1000 ($\overline{10}^3$), 1000000 ($\overline{100}^3$), *&c.* therefore the ſecond Radius on E muſt begin with either 10, 10000, 10000000, *&c.* and conſequently the third Radius will begin with either 100, 100000, 100000000, *&c.* Q. E. D.

Example.

What is the Cube Root of 3375?

Set the Line D (on the Slide) exactly even with the Line E; then, againſt 3375 on the firſt Radius on E, according to the preceding Obſervation, we have 15 on D, the required Root.

The Lines C and D are likewiſe very uſeful in finding a Mean-Proportional between any two given Numbers; alſo in finding, from any three given Numbers, a fourth, which ſhall be to the third, as the Square of the ſecond is to the Square of the firſt Number; and therefore theſe Lines are applicable to the finding the Areas of Circles, (which are as the Squares of their Diameters) and the Contents of ſuch Solids, whereof the Square of one Dimenſion, being multiplied into another Dimenſion, ſhall expreſs either the whole Content, (as in an upright ſquare Priſm) or ſome Multiple of it, as a *Cylinder*, *Cone*, *Sphere* and *Spheroid*; and conſequently, theſe Lines may be applied in finding the correſponding Dimenſions of ſimilar Surfaces.

The Lines D and E are neceſſary in finding, from any three given Numbers, a fourth Number, which ſhall be to the third, as the Cube of the ſecond is to the Cube of the firſt; and conſequently in determining the Contents of ſimilar Solids, which are in the direct Proportion of the Cubes of their correſponding Dimenſions; and likewiſe, on the contrary, in finding the correſponding Dimenſions of ſimilar Solids.

Prop. 3.

To find a Mean-Proportional between two given Numbers; or, which is the ſame Thing, to find the

the Square Root of the Product of any two given Numbers.

RULE.

Set one of the given Numbers on C, to the like Number on D; then againſt the other given Number on C, is the Mean-Proportional ſought on D.

EXAMPLE.

What is the Mean-Proportional between 4 and 9?

Set 4 on C, to 4 on D; then againſt 9 on C, is 6 on D, the Anſwer ſought.‡

PROP. 4.

To find, to any three given Numbers, a fourth Number, which ſhall be to the third, as the Square of the ſecond is to the Square of the firſt.

RULE.

To the firſt Number (or Root) on D, ſet the third on C; then againſt the ſecond Number (or Root) on D, is the fourth Number ſought on C.

EXAMPLE

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‡ By placing 4 on C, to 4 on D, we get the Sum of the Diſtances from 1 to 2 on the Line D, and alſo from 1 to 9 on the Line C; the former being $\frac{1}{2}$ the Diſtance of 1 to 4 on D, and the latter (being on the double Radius) is the ſame as 1 to 3, meaſured on the Line D; but theſe Diſtances are reſpectively as the Logarithms of 2 and 3; therefore the Log. 2 + Log. 3 ($= \frac{1}{2}$ L. 4 + $\frac{1}{2}$ L. 9) = Log. $\overline{4 \times 9)}^{\frac{1}{2}}$ = Log. 6.

EXAMPLE 1.

Suppoſe the given Numbers were 3, 9, and 12, and that it was required to find a fourth Number, which ſhall be in the ſame Proportion to 12, as the Square of 9 is to the Square of 3; that is, as 81 to 9.

To 3 on D, ſet 12 on C; then againſt 9 on D, is 108 on C, the Anſwer ſought.

EXAMPLE 2.

If 3 Feet of cylindrical dried Oak, whoſe Circumference is 32 Inches, weigh 80 lb. what will 3 Feet of the ſame Sort of Oak weigh, when the Circumference is 22 Inches?

The Altitudes of the two Cylinders being equal to each other, therefore their Solidities, and conſequently their Weights, muſt be to each other as the Areas of their Baſes; which are the Squares of their Diameters, or Circumferences.

To 32 on D, ſet 80 lb. on C; and againſt 22 on D, is 37.8 lb. on C, the Weight ſought.

If it ſhould happen, that, when the third Number on C is ſet to the firſt Number on D (according to the foregoing Rule), the ſecond Number on D falls beyond the Slide, or Line C; then, in ſuch Caſe, we need only to multiply, or divide (according as it falls off the Rule towards the Left or Right-Hand) the ſaid ſecond Number (or Root) by ſuch a Number, that the Product, or Quotient, thereof may be found on D, oppoſite ſome Number on C; which Number being divided, or multiplied, by the Square of that Number, by which the ſecond

was

was multiplied, or divided; the Quotient, or Product, will be the Anſwer ſought.

Suppoſe, in the firſt of the two preceding Examples, the ſecond Number was 15, and the other two the ſame as before. — To 3 on D, ſet 12 on C; then againſt 7.5 (the Half of 15) on D, is 75 on C, which being multiplied by 4 (the Square of 2) gives 300, the Anſwer ſought.

It will, in many Caſes, be moſt convenient to multiply, or divide, the ſecond Number by 10, and then find (as above directed) the Number on C, oppoſite that Product, or Quotient; which Number being divided, or multiplied, by 100 (the Square of 10) gives the Anſwer ſought.

The above Method renders the Buſineſs of finding what are called *new Gauge-points* quite unneceſſary, as ſhall be explained farther on.

PROP. 5.

Any three Numbers being given to find a fourth, ſo that the Square thereof ſhall be to the Square of the third, as the ſecond Number is to the firſt.

RULE.

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de

* Let the three given Numbers be noted by m, n, and r, and the Number ſought by v, and alſo *that* by which the ſecond Number is either multiplied or divided, be repreſented by d: Then, (by the *Prop.*) $m^2 : n^2 :: r : v$; but $m^2 : \frac{n}{d} \times \frac{n}{d} :: r : \frac{v}{d^2}$; alſo $m^2 : dn \times dn :: r : dv^2$; whence it is plain, that, when the ſecond Number (n) is only $\frac{n}{d}$, the fourth Number (v) will then only be the d^2 Part of *that*, when the ſecond Number is n; and likewiſe when the ſecond Number is $= d \times n$, the fourth Number, or Anſwer ſought, will then become d^2 times *that*, when the ſecond Number is equal only n. Q. E. I.

RULE.

To the third Number (or Root) on D, ſet the firſt Number on C; then againſt the ſecond on C, is the required fourth Number (or Root) on D.

EXAMPLE I.

Let the Side of a Triangle, whoſe Area is 15 Gallons, be 40 Inches; what will be the correſponding Side of another ſimilar Triangle, the Area of which is to be 60 Gallons.

Here the three given Numbers are 15, 60, and 40: Therefore, according to the above Rule, to 40 on D, ſet 15 on C; then, againſt 60 on C, is 80 on D, the required Side.

If, when the Rule is ſet as above, the ſecond Number on C falls off the Line D; then let the third Number be multiplied, or divided, (according as the ſecond Number on C falls off towards the Left or Right-Hand), by ſuch a Number, that, if to the Product, or Quotient, thereof, the firſt Number on C be ſet, the ſecond Number on C may fall oppoſite ſome Number on D; which being divided, or multiplied, by that Number with which the third was multiplied, or divided; the Quotient, or Product will be the Anſwer ſought.*

 EXAMPLE

* Let every Thing be interpreted as in the preceeding *Note*. Then we have (by the *Prop.*) $m : n :: r^2 : v^2$; but $m : n :: \frac{r}{d} \times \frac{r}{d} : \frac{v}{d} \times \frac{v}{d}$; likewiſe, $m : n :: dr \times dr : dv \times dv$; $\because$ it is evident, that, when the third Number is only $\frac{r}{d}$ (inſtead of r), the *fourth Number* (v) will be only the dth Part of *that* when the third Number is r: Moreover, when the third Number (r) is $= d \times r$, the *fourth Number* (v) will then be $= d$ times *that* when the third Number is only r. Q. E. I.

EXAMPLE 2.

Let the three given Numbers be 15, 90, and 60; required to find a fourth Number, so that the Square thereof shall be to the Square of 60, as 90 is to 15.

To 60 on D, set (according to the preceding Rule) 15 on C; then 90 on C, manifestly, falls beyond the Line D: But, if to 15 ($\frac{1}{4}$ of 60 the third Number) on D, be set 15 on C; then opposite 90 on C, we have 36.75 *(very nearly)* on the Line D; which being multiplied by 4 gives 147, the required Number, *nearly* — For 15 is to 90, as 3600 (the Square of 60) is to 21660 (the Square of 147), *nearly*.

PROP. 6.

Let there be any three Numbers given to find a fourth, which shall be to the third, as the Cube of the second is to the Cube of the first Number.

RULE.

Set the first given Number (or Root) on the Slide D, to the third Number on E; then opposite the second Number (or Root) on D, is the fourth Number required on E.

EXAMPLE.

Suppose the given Numbers to be 3, 6, and 18; it is required to find a fourth Number, which shall be to 18, as the Cube of 6 is to the Cube of 3; or as 216 to 27.

Set

Set 3 on the Slide D, to 18 on E; then against 6 on D, is 144 on E, the fourth Number sought.*

PROP. 7.

Given any three Numbers to find a fourth, the Cube whereof shall be to the Cube of the third, as the second Number is to the first.

RULE.

Set the third Number (or Root) on the Slide D, to the first Number on E; then opposite the second on E, is the fourth Number sought on D.

EAMPLE.

Suppose, in the Frustum of a Cone, there are given the bottom Diameter 40, the top Diameter 25, and the Altitude 30 Inches; it is required to find the Dimensions of another similar Frustum (that is, the Diameters and Altitude to remain in the above Proportion to one another), whose Content shall exceed the former 50 Wine Gallons.

The Content of the given Frustum (by the Methods given farther on) is 109.6 Wine Gallons; therefore the Content of the required similar

 Frustum

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* It is evident, that, by setting 1 on D, to 18 on E, we shall obtain the Sum of the Distances 1 to 18 on E, and 1 to 6 on the Line D; which last is equal to 3 times the Distance from 1 to 6 on E; but by moving the Slide (towards the Left-Hand) till 3 on D is opposite 18 on E, we thereby diminish the Sum of the two said Extensions by the Distance of 1 to 3 on D (answering to 3 times the Distance from 1 to 3 on E), and moreover get the Distance from 1 to 18 on E, *plus* 3 times the Distance from 1 to 6 on E, *minus* 3 times the Distance from 1 to 3 on E: But, by the Construction of the Lines, these Distances are respectively as the Log. of 18, 3 × Log. 6, and 3 × Log. 3 whence, by the Property of Logarithms, the Log. 18 + 3 × Log. 6 — × Log. 3 $\left(= \frac{18 \times \overline{6}^3}{\overline{3}^3} = 144\right)$ = Log. 144.

Frustum will be 159.6 Wine Gallons:—Then, by the preceding *Proposition*, the three given Numbers stand thus:

109.6, 159.6, 40, to find the bottom Diameter.
109.6, 159.6, 25, to find the top Diameter.
109.6, 159.6, 30, to find the Altitude.

Set 40 on the Slide D, to 109.6 on E; then against 159.6 on E, is 45.3 on D, *nearly*:— Set 25 on D, to 109.6 on E; then opposite 159.6 on E, is 28.4 on D, *nearly*: —Lastly, set 30 on D, to 109.6 on E; then against 159.6 on E, is 34 on D, *nearly*.

Hence the required Dimensions are 45.3 for the bottom Diameter; 28.4 for the top Diameter; and 34 Inches for the Altitude, *nearly*.

It may be proper to take Notice, that what has been already said (*Prop.* 4.) with Respect to the second Number falling off the Line C, holds equally good with Regard to the Lines D and E; only observe, here, to multiply, or divide, by the Cube (instead of the Square) of that Number by which the second was divided, or multiplied.

SECTION

SECTION V.

Of Geometrical Definitions *of Lines, Angles, Surfaces, and Solids.*

Definitions.

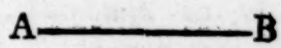
Of Lines and Angles.

1. A *Line* is Diſtance, or Length without Breadth; the Extremes, Bounds, or Limits of which are called *Points.*

Therefore,

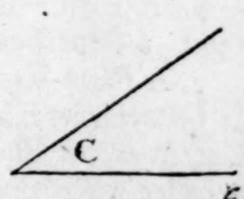
2. A *Mathematical Point* has no Parts.

3. A *Right-line* (or *Straight-line*) is that which lies perfectly even between its Extremes or Limits, as AB.

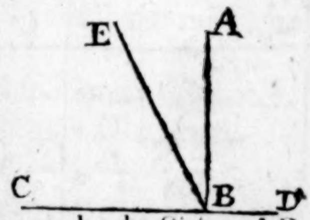


4. A *Curved-line* is that which, in every Part thereof, lies unevenly between its Extremes or Bounds, as *ab*.

5. A *Right-lined Angle* is that which is formed by the Inclination of two Right-lines, meeting each other in a Point, as C.

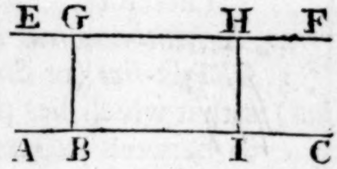


6. There are three Sorts of right-lined Angles. —— 1. When one Right-line AB ſtands any-where upon another CD, ſo as to incline no more towards one End than the other, making the Angles on both Sides AB equal, then thoſe Angles are called *Right-angles*; and the two Right-lines, AB and CD, are then ſaid to be perpendicular to each other. —2. When the Angle (EBD) is greater than a Right-angle (ABD), it is called an *Obtuſe-angle*. — 3. If the Angle (EBC) is leſs than a Right-angle (ABC), it is called an *Acute-angle*.



Note. When an Angle is denoted by three Letters (as ABC), that in the Middle ſtands at the angular Point, and the other two ſtand at the Extremities of the Lines which form the Angle: Thus, in the preceding Definition, the Letter B is the angular Point of the *Right*, *Obtuſe*, and the *Acute-angles*, there ſpecified.

7. Two Right-lines AC, EF are ſaid to be *parallel*, or *equidiſtant*, when Lines BG, IH drawn any where perpendicular to one of them AC, and terminating at the other EF, are equal to each other.



Of Planes, or Surfaces.

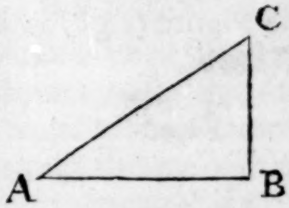
8. A *Figure* is the Form of either a Surface (*viz.* a Superficies), or a Solid.

9. A *Plane-ſurface* is any Figure which lies evenly between its Extremes, or Bounds; and if thoſe Extremes, or Bounds, are Right-lines, the Figure is called a *Rectilineal* (or *Right-lined*) *Plane*; but if the

the Extremes, or Bounds, of a Plane are crooked or Curve-lined, the Figure is then called a *Curvilineal Surface*, or *Plane*.

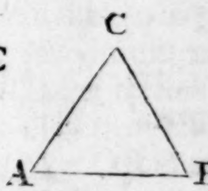
10. Every plane Figure, or Superficies, bounded by three Right-lines, is called a *Right-lined Triangle*.

11. A *Right-angled Triangle* ABC is that which has one Right-angle; the Sides AB, BC containing the Right-angle, are called the *Legs*, and the Side AC, opposite the Right-angle, is called the *Hypothenuse*.

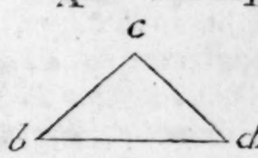


12. When the three Angles are all acute, it is called an *Acute-angled Triangle*; but if one Angle is obtuse, the Figure is then called an *Obtuse-angled Triangle*.

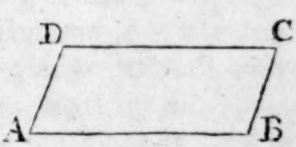
13. An *Equilateral Triangle* ABC has all its Sides equal.

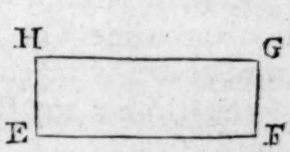


14. An *Isosceles Triangle* bcd has two of its Sides equal; and when the three Sides are all unequal, the Figure is called a *Scalene Triangle*.



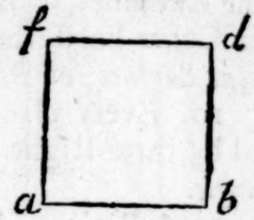
15. Every plane Figure, or Surface, bounded by four Right-lines, is called a *Quadrilateral*: Whereof, those (ABCD) whose opposite Sides are parallel, are called *Parallelograms*; and those (EFGH) whose opposite Sides are parallel, and all the Angles right-ones, are called *Rectangles*, or *Rectangular*





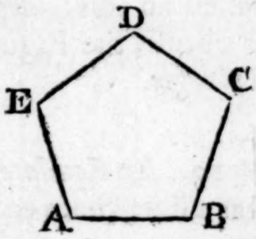
Parallelograms;

Parallelograms; and if the Sides, as well asthe Angles,are all equal, the Figure *(abdf)* is called a *Square:* — When the Sides are all equal, and only the opposite Angles equal, the Figure *(cegh)* is called a *Rhombus*,

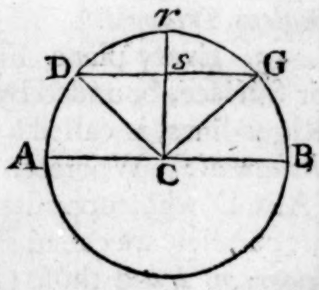


16. Every other plane Figure, bounded by four Right-lines, is called a *Trapezium.*

17. Any plane Figure, or Superficies, bounded by more than four Right-lines, is called a *Polygon*; and is named according to the Number of Sides it contains: Thus, if it has five Sides (ABC DE) it is called a *Pentagon*; if six Sides, a *Hexagon*; if seven, a *Heptagon*; if eight an *Octagon*, &c. If all the Sides and Angles are equal, as in the Figure ABCDE, it is called a *regular Polygon*; if otherwise, it is called an *irregular Polygon.*



18. A Circle is a plane Figure, bounded by one continued Line called the *Circumference*, or *Periphery*; every Part of which is equally distant from a Point within the Circle called its *Center*; from which, any Right-line (CA, CD, &c.) drawn to the Circumference, is called the *Radius*, or *Semi diameter* of the Circle; any Right-line AB drawn through the



the Center, terminating each Way at the Circumference, is called a *Diameter*; a Right-line DG less than the Diameter, meeting the Circumference in two Points, is called a *Chord*, or *Subtense*; and the perpendicular Distance *rs*, from the Middle of the Chord to the Circumference, is called a *Versed Sine.*

19. A *Segment* of a *Circle* D*r*G is a Figure bounded by a Part of the Circumference, and its Chord DG; when this last is equal to the Diameter of the Circle, the Figure is called a *Semi-circle*, as AD*r*GB.

20. A *Sector* of a *Circle* D*r*GC is a Figure contained by an Arch (or Arc) thereof, and two Semidiameters; when these two form a Right-angle, or the Arc becomes $\frac{1}{4}$th of the Circumference, the Figure is called a *Quadrant*, as AD*r*C (or C*r*GB, see the last Fig).

21. The Circumference of every Circle is supposed to be divided into 360 equal Parts, called *Degrees*, each Degree into 60 equal Parts, called *Minutes*, and each Minute into 60 equal Parts, called *Seconds*, *&c.*

22. Every plane Angle DCG (see the preceding Figure) is measured by an *Arch* of a *Circle*, contained between the two Lines which form the Angle, and described upon the angular Point as a Center; and the Quantity of the Angle is estimated from the Number of Degrees and Minutes, *&c.* which the said *Arch* contains.

OF SOLIDS.

23. A *Solid* is that which has three Dimensions, *viz.* *Length*, *Breadth*, and *Thickness*: — The Figure of a Solid may be conceived to be generated either by the Motion of a Plane (or Surface) in

ſome certain Direction, or by the Revolution of a Plane round ſome Line as an Axis.

24. The *Bounds*, or *Extremes*, of a Solid are either plane or curved Surfaces.

25. A *Priſm* is a Solid, whoſe two Ends are parallel Planes, of any rectilineal Form whatever; the Planes of the Sides of this Solid are Parallelograms; when theſe ſtand perpendicular to the Plane of the Baſe, the Figure is called an *Upright Priſm*; when they ſtand otherwiſe to the Baſe, the Figure is called an *Oblique Priſm*; if the two Ends are Parallelograms, the Solid (being then contained under ſix Parallelograms) is called a *Parallelopipedon*; if the ſix bounding Planes are all Rectangles, the Solid is called a *Rectangular Parallelopipedon*; and when the ſix bounding Planes are all Squares, the Solid is then called a *Cube*.

26. A *Cylinder* is a Solid, whoſe two Ends are equal and parallel Circles: This Solid may be conceived to be formed, or generated, either by the Rotation of a Rectangle ABED about one of its Sides DE as an Axis; or by the Motion of a Circle C*m*B*n*, in a Direction perpendicular to itſelf, to any aſſigned Altitude DE: This is called a *Right Cylinder*. But if the Circle be ſuppoſed to move parallel to itſelf, in any other rectilineal Direction, it will thereby generate an inclined Cylinder; but this Solid never occurs in the Subject of Gauging.

If the two equal Ends of the *Solid* are (inſtead of Circles) of any curvilineal Form whatever, *it* is in general called a *Cylindroid*; and is farther diſtinguiſhed according to the Figure of its Baſes: Thus, if the two Ends were two equal, ſimilar,

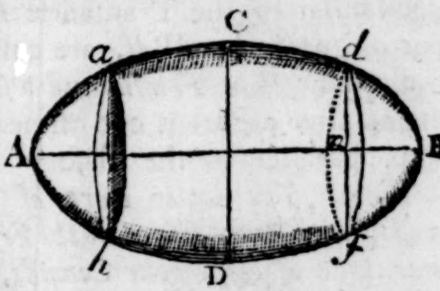
and

and similarly posited Ellipses; that is, the Transverse and Conjugate Axes of each End, respectively parallel to each other; the Solid is then called an *Elliptical Cylindroid*, &c.

27. A *Pyramid* is a Solid, whereof the Base is any right-lined Plane whatever; the Sides of this Solid are plane Triangles, whose vertical Angles (*i. e.* those opposite the Perimeter of the Base) all meet together in a Point above the Base, called the *Vertex* of the *Pyramid*.

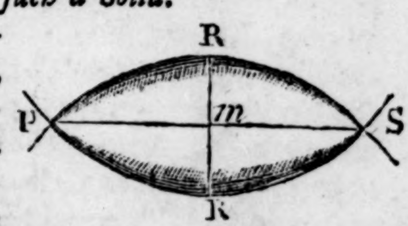
28. A *Sphere* is a Solid, generated by a Semicircle revolving about its Diameter as an Axis.

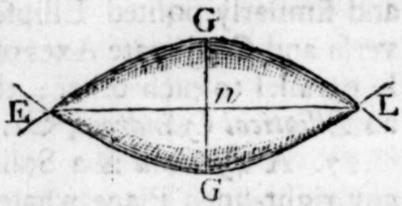
29. A *Spheroid* is a Solid, generated by the Rotation of a Semi-ellipsis about its Diameter as an Axis; if the Rotation be about the Transverse Axis AB, the generated Solid ACBD, is called an *Oblong Spheroid*; but if the Solid be generated about the Conjugate Axis CD, it is called an *Oblate Spheroid*.



Note. *The former of the two last mentioned Solids, is only applicable to the present Subject: Every* Spheroidical Cask, *that occurs in Practice, resembles the middle Frustum of such a Solid.*

30. A *Parabolic Spindle* is a Solid PRSR, generated by the Rotation of a Parabola PRS*m* about its Ordinate PS: But if the parabolic Space R*m*S (R*m*P) was to revolve about its Axis R*m*, it would thereby generate a Solid, called a *Parabolic Conoid*.





31. An *Hyperbolic Spindle* is a Solid EGLG, generated by the Rotation of an Hyperbola EGLn about its Ordinate EL: But when the hyperbolic *Space* GnL (GnE) revolves about its Axis Gn, it thereby generates the Resemblance of a Solid, called an *Hyperbolic Conoid*.

32. The *Middle Frustum* of a Spheroid DbaCdf (*see Fig. to Defin.* 29.) is what remains after two equal Parts are cut off, by Planes perpendicular to the Transverse Axis: The Parts so cut off, as Aab and Bdf, are called *Segments* of the Spheroid. The *Frustum* of a *Cone*,* is what remains after a Part is cut off next the Vertex, by a Plane parallel to the Base of the Cone.

Note. *The latter Part of this Definition extends, with equal Force, to the Frustums of Pyramids, Parabolic or Hyperbolic Conoids, and Spindles.*

* See the Definition of a Cone in the following Page.

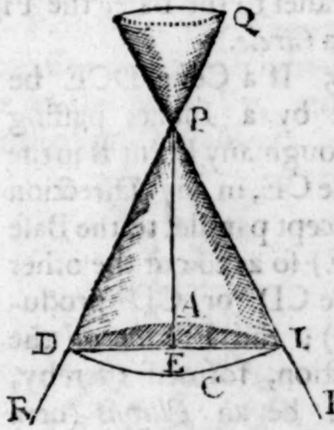
SECTION

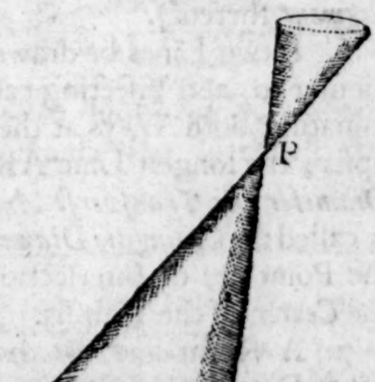
SECTION VI.

OF THE DEFINITIONS, AND SOME OF THE PRINCIPAL PROPERTIES OF CONIC SECTIONS.

DEFINITIONS.

1. IN an indefinite Right-line QR, conceive an immoveable (or fixed) Point P; upon which, as a Center, let the ſaid Line be moved juſt round, continually touching the Circumference of a Circle DAIC, placed in any Poſition (except in that of a Plane paſſing through the ſaid fixed Point); then that Part of the Line intercepted between the fixed Point and the Periphery of the Circle, will (by its Rotation) generate the convex Superficies of a Figure called a *Cone:* If the Axis PE, or the Line joining the fixed Point and the Center of the Circle, be perpendicular to the Plane thereof,





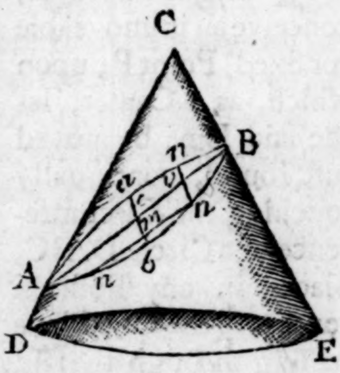
the Superficies then described, will be that of a *right Cone*, as DPI; otherwise it will be an *oblique* or *scalene Cone*, as GPL.

2. The Line PQ (*Fig.* I.) on the contrary Side of the fixed Point P, will also generate the convex Surface of another similar Cone; and these together are called *Opposite Cones*.

3. If a right Cone DPI be cut into two equal Parts, by a Plane perpendicular to that of the Base; the Figure of the Section will be a right-lined *isosceles Triangle*.

4. If a Cone be cut into two Parts, by a Plane parallel to the Base, the Figure of the Section will be a *Circle*.

5. If a Cone DCE be cut by a Plane, passing through any Point B in the Side CE, in any Direction (except parallel to the Base DE) so as to cut the other Side CD (or CD produced); the Figure of the Section, formed thereby, will be an *Ellipsis* (or a *Segment* thereof).

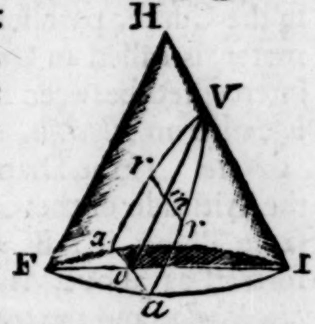


6. If two Lines be drawn in this Figure perpendicular to, and bisecting each other, and each terminating both Ways at the Periphery of the Ellipsis; the longest Line AB is called the *Transverse Diameter* (or *Transverse Axis*), and the shortest *ab*, is called the *Conjugate Diameter* (or *Conjugate Axis*); the Point (*c*) of Intersection of these two Lines, is the *Center* of the Ellipsis.

7. A Right-line *nn* drawn perpendicular to either Diameter, terminating both Ways at the Periphery of the Ellipsis, is called an *Ordinate* to that Axis which it intersects; and the Distance *v*B (*v*A,

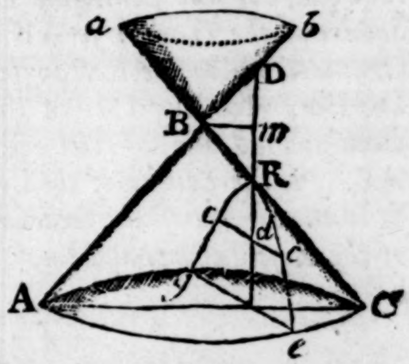
(*v*A, *mb* or *ma)* in the Axis, from the Ordinate to the Vertex B (A, *a* or *b)*, is called an *Abſciſſa.*

8. If a Cone FHI be cut into two Parts, by a Plane, in a Direction parallel to the ſlant Side thereof; the Figure of the Section *a*V*av* is called a *Parabola.*



9. The Right-line V*v* drawn from the Vertex V, parallel to the ſlant Side of the Cone, dividing the Area of the parabolic Section into two equal Parts, is called the *Axis* of the Parabola; any determinate Part from the Vertex V (as V*m)* is called the Abſciſſa; and any Right-line *mr* drawn perpendicular to the Axis V*v*, terminating at the Curve, is called an *Ordinate.*†

10. If a Cone ABC be cut into two Parts by a Plane, which, being continued, would alſo cut the oppoſite Cone *a*B*b*; the Figure of the Section R*eg* is called an *Hyperbola*;-- The Diſtance DR, intercepted between the two oppoſite Cones, is called the *Tranſverſe Diameter* (or *Axis)*, and the Diſtance B*m* is called the *Semi-*



† Though an Ordinate, ſtrictly ſpeaking, is that Line in a Conic Section, which is biſected by the Axis (or Diameter), terminating each Way at the Curve; yet Geometricians frequently call the Half of this Line (or the Diſtance from the Curve to the Axis, or Diameter) the *Ordinate:* For the general Property of the Curve is the very ſame in both Caſes; becauſe the Squares (or any Power or Multiple) of the Wholes, are in the ſame Ratio, as the Squares (or the ſame Power or Multiple) of their Halves.

Semi-conjugate Diameter (or *Axis*): Moreover, the Right-line *cd*, drawn from any Point *c* in the Curve, parallel to the Semi-conjugate Diameter, is called an *Ordinate*; and the Diſtance R*d*, intercepted between that Ordinate and the Vertex, is called an *Abſciſſa*.

Note. If the Diameter of the Baſe be double the Altitude of the Cone; or, which comes to the ſame Thing, if ABC is a Right-angle, the Section, formed as above, is then called an *Equilateral Hyperbola*, and the two Diameters DR and twice B*m* become equal to each other.

PROP. I.

The general Property of every Ellipſis will be, as the Square of the Conjugate Diameter a b, is to the Square of the Tranſverſe AB; *ſo is the Square of the Ordinate vn, to the Rectangle of the Abſciſſas* A*v and* B*v*; (ſee Fig III. of the preceeding Definitions): *And alſo, as the Square of the Tranſverſe* AB, *is to the Square of the Conjugate Diameter a b; ſo is the Square of the Ordinate mn, to the Rectangle of the Abſciſſas a m and bm.*†

PROP.

---

† Let DE (*Fig.* I. *in the Plate*) be the Tranſverſe, and TH the Conjugate Diameter of an Ellipſis; through the Center M, and alſo the Point of Interſection (*m*) of the Ordinate (*bb*) with the Tranſverſe Diameter, draw RS and GK, each parallel to the Diameter of the Cone's Baſe: Then (ſuppoſing ACF to be the Plane paſſing through the Vertex and the Center of the Baſe of the Cone), by ſimilar Triangles, DM : RM :: D*m* : G*m*; again, by ſimilar Triangles, EM : SM :: E*m* : K*m*; whence, by multiplying the Antecedents and Conſequents of both Proportions by each other, we get $DM \times EM : RM \times SM :: Dm \times Em : Gm \times Km$; but, by the Property of the Circle, $RM \times SM = MH^2$, and alſo $Gm \times Km = bm^2$; ∴ $DM \times EM : MH^2 :: Dm \times Em : bm^2$; but DM = EM; ∴ $HM^2 : DM^2$ (EM^2) :: $bm^2 : Dm \times Em$; or, which is the ſame Thing, TH^2 ($4 \times MH^2$) : DE^2 ($4 \times DM^2$) :: $bm^2 : Dm \times Em$. Q. E. D.

Draw

PROP. II.

The general Property of every conic Parabola (see Fig. IV. of the foregoing *Def.*) *will be, that the Squares of any two Ordinates, r m and a v, are to each other as their corresponding Abscissas* V*m and* V*v*.†

PROP. III.

The general Property of the Hyperbola (see *Fig.* V. of the last *Def.*) *will be, as the Square of any Ordinate d c, to the Transverse Diameter* DR, *is to the Rectangle contained under the corresponding Abscissa* R*d*, *and the Sum of that Abscissa and the Transverse Diameter*, viz. D*d*; *so is the Square of the Conjugate Diameter twice* B*m*, *to the Square of the Transverse* DR.‡

K The

Draw the Ordinate bn; then, by the last Proportion, we have $bm^2 : EM^2 - Mm^2$ ($EM^2 - bn^2$, or $\overline{EM+Mm} \times \overline{EM(ML)-Mm}$) $:: TH^2 : DE^2$, that is, $bm^2 \times DE^2 = \overline{EM^2 - bn^2} \times TH^2$, or $bn^2 \times TH^2 = EM^2 \times TH^2 - bm^2 \times DE^2$; but $EM^2 \times TH^2 - bm^2 \times DE^2 = 4EM^2 \times TM^2 - bm^2 (Mn^2) \times DE^2$ (or $4DM^2$); whence $bn^2 \times TH^2 = \overline{TM^2 - Mn^2} \times 4EM^2$ ($= \overline{TM+Mn} \times \overline{TM(MH)-Mn} \times 4EM^2$); that is, $TH^2 : DE^2 (4EM^2) :: TM^2 - Mn^2 (Tn \times Hn) : bn^2$. Q. E. D.

† Draw DmG (Fig. II. in the Plate) parallel to the Diameter FI; then the Triangles vVI and mVG are similar, and ∵ $Vv : vI :: mV : mG$; or $Vv \times mG = Vm \times vI$; but, because $Dm = Fv$, $Vv \times mG \times Dm = Vm \times vI \times Fv$; and by the Property of the Circle, $mG \times Dm = mr^2$, and also $vI \times Fv = va^2$; which being substituted above, we get $Vv \times mr^2 = Vm \times va^2$; that is, $Vv : Vm :: va^2 : mr^2$. Q. E. D.

‡ Let Dra be parallel to Bm the Axis of the Cone, through ra the Axis of the Section, draw MdI (*Fig.* III.) parallel to the Semi-conjugate Bn, and also let the Ordinate cc (lying in the same Plane with MdI) be drawn: Then, by similar Triangles, $Dn : Bn :: Dd : Id$; again, by similar Triangles, $nr : Bn :: dr : dM$; ∵ $Dn \times nr : Bn^2 :: Dd \times dr : Id \times dM$; but $Dn = nr$; ∵ $nr^2 : Bn^2 :: Dd \times dr : Id \times dM$; moreover, by the Property of the Circle, $cd^2 = Id \times dM$; whence we have $nr^2 : Bn^2 :: Dd \times dr : cd^2$, or $Dr^2 (4nr^2) : \overline{2Bn}^2 (4Bn^2) :: Dd \times dr : cd^2$. Q. E. D.

The foregoing *Definitions* and *Properties* of the Sections of a Cone are absolutely necessary to be well understood by every practical Gauger, who would clearly apprehend what is meant by the Words, *Ellipsis*, *Parabola*, and *Hyperbola*; and also by the Solids, which may be conceived to be generated by the Rotation of those *Curves* about their Diameters, or Ordinates: Such as the *Spheroid*, *Parabolic Spindle*, and *Hyperbolic Spindle*, *&c.* from whence the three Varieties of Casks are formed.

SECTION VII.

Of the Measure of plane Figures; *or of finding their Areas* in Ale and Wine Gallons, and Malt Bushels.*

*THE Area, or Measure, of any plane Surface, Geometrically considered, is the whole Space contained under the Bounds of the Figure, without any Regard to Thickness; as in the Mensuration of *Land*, *Painter's Work*, *&c.* This Area, or superficial Content of the Space, is computed from another Space of a determinate Form and Magnitude; that is, from a *Square* whose Side is one *Inch*, *Foot*, *Yard*, *&c.* called the *measuring Unit*; and the Number of such *Squares* or *Units*, (and Parts of an Unit), that are contained in any plane Figure, is called the *Content*, or *Measure*, of that Figure. But, in the Subject of Gauging, where the *measuring Unit* is one Ale or Wine Gallon, or Malt Bushel, it will be most commodious, in

in Order to express the Area of a plane Figure in such Denominations, to consider the Plane (or rather *Solid*) to be just one Inch thick; by which Means, if the Number that expresses the square Inches contained in any plane Figure, be divided by the Number expressing the cubic Inches in the Ale or Wine Gallon, or Malt Bushel, we shall obtain the Area, or the Content of the Figure, in those Denominations.

Though it is said (above), that any plane Figure will contain a Number of little *equal Squares*, yet it is not to be understood that all plane Figures can be *formed* with a Number of such Squares; but because a Square (which can be so formed) may be found, whose Area shall be exactly (or *nearly*) equal to that of any plane Figure whatever.

The very same is to be observed, with Respect to every solid Figure (except a *Cube*, or a rectangular *Parallelopipedon*) not being formed with a Number of little *equal Cubes*.

Note. A Gallon of Ale, (*Beer*, or *Vinegar*) contains 282 cubic Inches.

A Gallon of Wine (*Sweets*, *Cyder*, *Perry*, *Verjuice*, *Wash*, *Low-Wines*, *Spirits*, &c.) contains 231 cubic Inches.

A *Winchester* Bushel of Malt contains 2150.42 cubic Inches.

				Gall.	
In 1 Barrel	*London*	Beer	are	36	each Gallon 282 cubic Inches.
		Ale		32	
	Country	Beer & Ale		34	
		Vinegar		34	
	1 Barrel of	Sweets		$31\frac{1}{2}$	Wine Measure.

PROP. I.

To find the Area of any plane Triangle in Ale and Wine Gallons, and Malt Bushels.

 RULE.

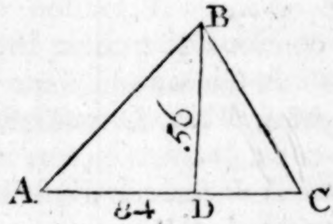
RULE.

From any one of the given Angles let fall a Perpendicular upon the Side opposite (produced if needful); multiply this Side, taken in Inches, by half the Perpendicular, taken in the same Measure, the Product will be the Area of the Triangle in square Inches; which being divided by 282 for Ale, 231 for Wine Gallons, and by 2150.42 for Malt Bushels, the Quotient will be the required Area of the Triangle.

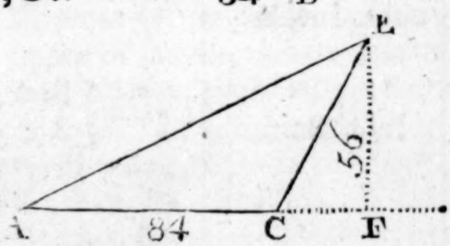
Note. The Measure, or superficial Content, of any plane Triangle is likewise obtained, by multiplying the whole Perpendicular by half the Base; or by taking half the Product contained under the whole Base and Perpendicular.

EXAMPLE.

The Base AC of the Triangle ABC (or AEC) is 84, and the Perpendicular BD (or EF) is 56 Inches; required the Area in Ale Gallons, *&c.*



OPERATION.



Base AC 84
½ Perp. BD (EF) 28

672
168

Product 2352, the Area of the Triangle ABC (AEC) in Inches.

282)

282)2352.00(8.34, the Area in Ale Gallons.
231)2352.00(10.18, the Area in Wine Gallons.
2150.42)2352.00(1.09, the Area in Malt Bushels.

By the Sliding-Rule.

$$\text{To}\left\{\begin{array}{l}282\\231\\2150.42\end{array}\right\}$$ on A, set 28 (or 84) on B; then opposite 84 (or 28) on the first Radius on A, is the above Areas respectively on B.*

Note. The above Areas may be obtained by the Lines D and C, but not without extracting the Square Root, which would render the Operation more troublesome than that above exhibited, by the Lines A and B.

PROP. II.

To find the Area of a Square afdb in Ale and Wine Gallons, and Malt Bushels. (See Defin. 15, P. 56).

RULE.

* It is evident that the Characteristics (or Indices) in Logarithms, answer to the Number of Radii of the Rule; that is, any Number greater than 1 and less than 10, where the Characteristic is = 0, is found on the first Radius; likewise any Number greater than 10 and less than 100, the Characteristic being = 1, must be found (according to the *true* Numeration of the Lines) on the second Radius, *&c.* For the Logarithm of any Number is the very same as the Log. of 10, 100, 1000, *&c.* times that Number, except in the Characteristics, which are = the Exponents of the Powers of 10: Therefore, in the preceding Example, the Log. 2.8 + Log. 8.4 — Log 2.82 (= Log. 28 + Log. 84 — Log. 282) = Log. 8.34: Whence it appears, that by setting 2.8 on B to 1 on A, we shall get the Distance from 1 to 2.8 on B, and from 1 to 8.4 on A in one Sum, measured on the Slide B; but this Distance must evidently be diminished by *that* denoting the Log. of 2.82; to effect which, move the Slide towards the Right-Hand, till 2.8 on B, stands opposite 2.82 on A; then against 8.4 on A, is 8.34 on B; the same as before.

RULE.

Multiply the Side of the Square by itſelf, and divide the Product by 282 for Ale, 231 for Wine Gallons, and by 2150.42 for Malt Buſhels; or (which will be exact enough in Practice) by 2150.

EXAMPLE

The Side of the Square *afdb* being 50 Inches; required its Area in Ale Gallons, *&c.*

OPERATION.

$$\begin{array}{r} 50 \\ 50 \\ \hline \end{array}$$

Product 2500 the Area in Inches: Then
282)2500.00(8.86, the Area in Ale Gallons.
231)2500.00(10.82, the Area in Wine Gallons.
2150)2500.00(1.16, the Area in Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{array}{c} 16.79 \\ 15.19 \\ 46.37 \end{array}\right\}$ on D, ſet 1 on C; then oppoſite 50 on D, is $\left\{\begin{array}{c} 8.86 \\ 10.82 \\ 1.16 \end{array}\right\}$ on C; the ſame as above.

Otherwiſe, by the Sliding-Rule.

To $\left\{\begin{array}{c} 282 \\ 231 \\ 2150 \end{array}\right\}$ on A, ſet 50 on B; then againſt 50 on A, is the above Areas reſpectively on B.

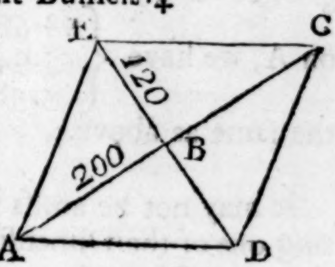
PROP.

PROP. III.

To determine the Area of a Rhombus in Ale and Wine Gallons, and Malt Bushels.

RULE.

Multiply the two Diagonals together: Half that Product divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Bushels.‡



EXAMPLE.

Suppose the Diagonal ED 120, and AC 200 Inches; required the Area of the Rhombus in Ale Gallons, *&c.*

OPERATION.

120
200

The Product of the Diag. 24000

Half the Product is 12000 equal to the Area in Inches.

282)

‡ By Reason of the equal and parallel Lines, it is plain *(Eu.* 5. & 29. 1.) that the opposite Angles are all bisected by the Diagonals; consequently the Rhombus (see the above *Fig.)* is evidently divided (by the Diagonals) into four right-angled Triangles, similar and equal in every Respect; the Area of any one of which will be expressed by AB $\times \frac{BD}{2}$, or $\frac{AC}{2} \times \frac{ED}{4}$; consequently the Area of the whole Rhombus $= \frac{AC \times ED}{2}$, Q. E. D.

282)12000.00(42.55 equal to the Area in Ale Gallons.

231)12000.00(51.94 = the Area in Wine Gallons.

2150)12000.00(5.58 = the Area in Malt Buſhels.

By the Sliding-Rule.

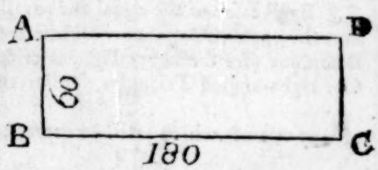
To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, ſet 120 on B; then againſt 100 on A, we have $\left\{\begin{array}{c}42.55\\51.94\\5.58\end{array}\right\}$ the required Areas on B; the ſame as above.

It may not be amiſs to obſerve here, that by ſetting one of the Dimenſions on B, to 2150 on A, the other Dimenſion cannot be found on the Line A; unleſs the Rule conſiſts of more than two Radius's, or the ſaid Dimenſion is equal to, or greater than 100: We muſt therefore, in ſuch Caſes, have Recourſe to the Method already delivered in *Page* 42.

PROP. IV.

To find the Area of a Parallelogram in Ale and Wine Gallons, and Malt Buſhels.

RULE.

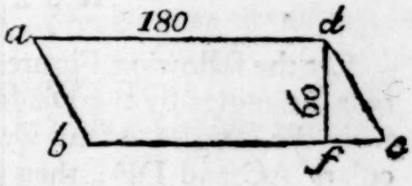


If it is a *rectangular* Parallelogram, as ABCD: Multiply the longeſt Side by the ſhorteſt; if otherwiſe, as *abcd* (called by ſome a *Rhomboides*); then multiply the longeſt Side *bc*,

bc, by the Perpendicular *df*, and divide the Product by 282, 231, and 2150; the Quotients will be the Areas in Ale and Wine Gallons, and Malt Bushels respectively.

EXAMPLE.

Suppose the longest Side BC (or *bc*) 180, the shortest Side AB (or the Perpendicular *df*) 60 Inches; required the Area in Ale Gallons, &c.



OPERATION.

180
60

Product is 10800, equal the Area in Inches.

282)10800.00(38.3 *nearly*, = the Area in Ale [Gallons.

231)10800.00(46.75 = the Area in Wine Gal- [lons.

2150)10800.00(5.02 = the Area in Malt Bushels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, set either of the above given Dimensions on B; then opposite the other Dimension on A, is $\left\{\begin{array}{c}38.3\\46.75\\5.02\end{array}\right\}$ on B; the same as above.

Prop. V.

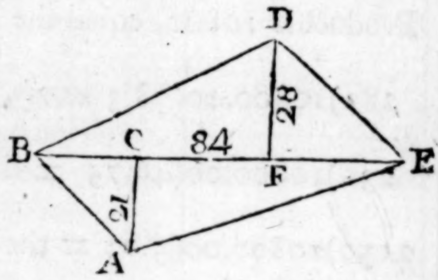
To find the Area of a Trapezium in Ale and Wine Gallons, and Malt Bushels.

Rule.

Let the following Figure ABDE be divided into two Triangles by the Diagonal EB; upon which, from the Angles A and D, let fall the Perpendiculars AC and DF; then multiply the Diagonal BE by half the Sum of those Perpendiculars; or their Sum by half the Diagonal, and divide the Product by 282 for Ale, 231 for Wine, and 2150 for Malt Bushels.

Example.

Suppose the Diagonal BE to be 84 Inches, the Perpendiculars AC and DF to be 21 and 28 Inches respectively; required the Area in Ale Gallons, *&c.*



Operation.

28
21

Sum of the Perp. 49
½ the Diagonal (84) is 42

98
196

Product is 2058, the Area in Inches.

282)

282)2058.0(7.3 *nearly*, the Area in Ale Gallons.
231)2058.0(8.9 Wine Gallons.
2150)2058.00(.95 Malt Bushels.

By the Sliding-Rule.

To $\left\{\begin{matrix}282\\231\\2150\end{matrix}\right\}$ on A, set 49 on B; then opposite 42 on A, we have $\left\{\begin{matrix}7.3\\8.9\\0.95\end{matrix}\right\}$ on B; the same as above.

By the preceding Method of dividing the Trapezium, the Measure of any irregular Polygon may be very easily obtained: For if the whole Figure be divided into Triangles, and the Area of each of those be found (*by Prop* I.); then will the Sum of those Areas be the Measure of the whole Polygon.

PROP. VI.

To find the Area of any regular Polygon in Ale and Wine Gallons, and Malt Bushels.

RULE.

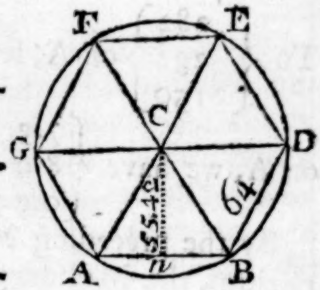
Half the Sum of all the Sides, being multiplied by a Line drawn from the Middle of any one of the Sides to the Center of the Polygon (or the Circle circumscribing it), and the Product divided by 282, 231, and 2150; the Quotients will be the Areas in Ale and Wine Gallons, and Malt Bushels respectively.

EXAMPLE.

In the Hexagon ABDEFG, if one of its Sides AB (BD, &c.) be 64 Inches, the Perpendicular

C*n* (C*r*, &c.) will be 55.42 Inches; required the Area in Ale Gallons, &c.

OPERATION.



$$\begin{array}{rr}
 & 64 \\
 & 6 \\
\hline
 & 384 \\
\hline
\tfrac{1}{2}\text{ the Sum of the Sides} & 192 \\
\text{Perpendicular } Cn & 55.42 \\
\hline
 & 384 \\
 & 768 \\
 & 960 \\
 & 960 \\
\hline
\end{array}$$

Product 10640.64, the Area in Inches.
282)10640.64(37.73 = the Area in Ale Gallons.
231)10640.64(46.06 Wine Gallons.
2150)10640.64(49.4 Malt Bushels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, set 192 on B; then against 55.4 on A, we have $\left\{\begin{array}{c}37.73\\46.06\\4.94\end{array}\right\}$ on B; the same as above.

Any regular Polygon is composed of as many *isosceles* Triangles as it contains Sides, and may be inscribed in a Circle, whose Center is that of the Polygon's; whence the *(equal)* Angles at the Center become known: And therefore it follows, that, if the Measure of the Side of any regular Polygon be one Inch, the Perpendicular C*n*

C*n* (see the laſt Figure) and the Area of the Polygon (in Inches) become known; which being divided by the proper Diviſors, the Quotients will give the reſpective Multipliers* for Ale and Wine Gallons, and Malt Buſhels:— Theſe Multipliers, conformable to all Authors on this Subject, I have exhibited in the following Table, for ſix different Kinds of regular Polygons. Now it is well known, to Geometricians, that the Areas of ſimilar (or like) plane Figures, are in Proportion to one another, as the Squares of the correſponding Sides; therefore, having obtained (as above) the Area of a Polygon whoſe Side is Unity, we then ſay, by Proportion, as the Square of 1 (which is 1), is to the Square of the Side of the Polygon whoſe Area is ſought; ſo is the Area of that Polygon whoſe Side is Unity (expreſſed in the following Table), to the Area of the Polygon ſought: Hence the following

RULE.

---

* Theſe Multipliers, or Factors, may be otherwiſe derived; by ſuppoſing (inſtead of the Side) the Radius of the Circle circumſcribing the Polygon $= 1$: For if a denote the Number of Sides of a Polygon circumſcribing that Circle, and t the Tangent of $\frac{1}{2}$ the Angle at the Center; which Angle is always known, from the Number of Sides of the circumſcribing Polygon: Then the Area of the Polygon circumſcribing the Circle, whoſe Radius is $= 1$, will be expreſſed by $1 \times t \times a$ or ta, and the Square of one of its Sides by $4t^2$; $\because 4t^2 : ta :: r^2$ (the Square of the Side of any other Polygon) : its Area; but $\frac{ta}{4t^2}$ (or $\frac{a}{4t}$) is $=$ the tabular Number, or Factor (ſee *Shirtcliffe*'s *Gauging*, P. 68), and is therefore $=$ the Area of a Polygon, whoſe Side is Unity, and Number of Sides $= a$: Which is thus approved:—Let m repreſent the Sine of an Angle whoſe Tangent is t; then, by ſimilar Triangles, $t : 1$ (Radius) $:: m : \frac{m}{t} =$ the Coſine; $\because \frac{m^2 \times a}{t} =$ the Area of the Polygon whoſe Side is $2m$, and Number of Sides a; whence, by ſimilar Figures, $4m^2 : \frac{m^2 \times a}{t} :: \overline{1}|^2 : \frac{m^2 \times a}{4m^2 \times t} = \frac{a}{4t}$. Q. E. I.

RULE.

Multiply the Square of the Side of a given regular Polygon by ſuch a Number, taken out of the following Table, as is agreeable to the Name of the Polygon; and the Product will be the Area thereof, in the ſame Denomination as the Factor that was made Uſe of.

A TABLE *for finding the* AREAS *of regular* POLYGONS.

The Name of the Polygon.	The Numb. of Sides.	The Angle at the Center.	The Area in Inches when the Side of the Polygon is unity	The Area in Ale Gallons.	The Area in Wine Gallons.	The Area in Malt Buſhels.
Pentagon	5	72° 0′	1.7204	.006099	.007445	.000780
Hexagon	6	60 0	2.5980	.009212	.011246	.001208
Heptagon	7	51 $25\frac{5}{7}$	3.6339	.012883	.015727	.001689
Octagon	8	45 0	4.8284	.017120	.020900	.002245
Nonagon	9	40 0	6.1818	.021925	.026726	.002875
Decagon	10	36 0	7.6942	.027287	.033311	.003579

In the preceding Example the Side of the Hexagon is 64 Inches; the Square whereof being multiplied by 2.598, the tabular Number for that Figure, gives 10641.4, the Area in Inches; the ſame as in *Page* 76, *very nearly.*

Having ſhewn the Methods of computing the Areas of ſuch right-lined Planes, as chiefly occur in the Practice of Gauging; I ſhall now proceed to determine the Areas of curvilineal Planes; as the Circle, Ellipſis, and their Segments, *&c.* But, firſt of all, it will be very neceſſary to ſhew the Learner, how to find the Circumference of a Circle, by having its Diameter given, and the contrary.

It

It is now looked upon, even by Mathematicians of the first Rank, as absolutely impossible to determine the *exact* Proportion of the Diameter and Circumference of a Circle.

That *great* Geometer *Archimedes*, about two Thousand Years ago, first discovered this Proportion to be nearly as 7 to 22; that is, if the Diameter of a Circle be 7, its Circumference will be 22, *very nearly*.

Since *Archimedes*'s Time, various Methods have been invented, whereby the said Proportion may be approximated to a very great Degree of Exactness. *Van Ceulen* (a *Dutch* Man) found, by incredible Pains, that if the Diameter of a Circle be represented by 1, the Circumference thereof will be 3.14159265358979323846264338327950288, *extremely near*; for if the last Decimal Figure be supposed 9, the said Number (3.1425, *&c.*) would then exceed the *true* Circumference of a Circle whose Diameter is 1: — This last Number was not only confirmed, but extended to double the Number of Decimal Places, by that ingenious and most indefatigable Mathematician, the late Mr. *Abr. Sharp* of *Little Horton*, near *Bradford*, in *Yorkshire*.

But, in the ordinary Practice of Gauging, it will be unnecessary to take any more than 3.14159 (or 3.1416): Hence it is evident, that if the Diameter of any Circle be multiplied by 3.14159, the Product will be the Circumference of that Circle, *very nearly*.

EXAMPLE.

To find the Circumference of a Circle, whose Diameter is 40 Inches.

OPERATION.

OPERATION.

$$\begin{array}{r} 3.1416 \\ 40 \\ \hline \end{array}$$

Product 125.6640 = the Circumference in [Inches.

By the Sliding-Rule.

Set 1 on B, to 3.1416 on A; then against 40 on B, is 125.6 on A.

It is evident, from this Example, that if the Circumference of any Circle be divided by 3.1416, the Quotient will be the Diameter thereof, *very nearly.*

If the Diameter and Circumference of a Circle are known, its Area will be found by multiplying half the Circumference by half the Diameter.*

But since the Areas of Circles (as well as all other similar plane Figures) are in Proportion to one another, as the Squares of their Diameters (or like Dimensions); it follows, that, if we have the Area of a Circle whose Diameter is Unity, we can easily obtain the Area of any Circle, whose Diameter is given, without finding its Circumference at all: Suppose, for Example, the Diameter of a Circle to be 1; then the Circumference, by the

* This evidently follows from the Rule given for regular Polygons, *Page* 75: For, since that Rule is general, let the Number of equal Sides be what it will; it follows, by conceiving a Polygon of an *indefinite* Number of Sides, that the Perpendicular C*n* (see Fig. P. 76) will then become the Radius of the Circle circumscribing that Polygon *indefinitely near*; and consequently the Perimeter of such a Polygon is, *very nearly*, equal to the Periphery of its circumscribing Circle: Whence it is evident, that the Measure of any Circle is equal to a Rectangle contained under half its Periphery and half its Diameter.

the aforesaid Proportion, will be 3.1416, *very nearly*; therefore, by the preceding Rule, we have the following

OPERATION.

3.1416

½ the Circumference	1.5708
½ the Diameter	.5

Product is .78540, the Area of a Circle whose Diameter is 1, *nearly*.

Therefore, if the Square of the Diameter of any Circle be multiplied by .7854, the Product will be the Area, or Measure, of the Circle in that Denomination whereby the Diameter was expressed, whether *Inches*, *Feet*, *Yards*, *&c.*

As, for Instance, suppose the Diameter of a Circle be 30 Inches, the Square whereof is 900; this being multiplied by .7854, gives 706.86 square Inches, the Area sought, *nearly*.

Now if the Area of any Circle in Inches (and in all other Figures) be divided by the Number of cubic Inches contained in a Gallon of Ale or Wine, *&c.* we shall obtain the Area of the Circle in those Denominations: But, in Order to avoid the above troublesome Multiplier (.7854), in finding the Area of a Circle in Ale or Wine Gallons, or Malt Bushels, we need only to square the Diameter, and multiply that by the Quotient of .7854 divided by the respective Divisors; or else divide the Square of the given Diameter, by the Quotients of the respective Divisors for Squares divided by .7854; and the Products, or Quotients, will be the Area of a Circle in the same Denomination as that of the Factor, or Divisor, used

Divisors for Squares, &c.	Factors for Circles, &c.	
282) .785398 &c.	(.0027850998 &c.	Ale Gal.
231) .785398 &c.	(.003399992 &c.	Wine Gal.
2150.42) .785398 &c.	(.0003652 &c.	Malt Bush.

Divisors for Circles.		
.785398)282.00 &c.	(359.05 &c.	Ale Gallons.
.785398)231.00 &c.	(294.118 &c.	Wine Gal.
.785398)2150.42 &c.	(2738.0003 &c.	M.Bush.

After the very same Manner may the Factors and Divisors be found, for obtaining the Areas of Circles in any other Denominations; which, for the Sake of Brevity, I shall exhibit in the following Table.

The Factors, or Multipliers, for reducing the Measures of Squares (or any Figure whatever) in Inches, into Ale and Wine Gallons, and Malt Bushels, are very easily obtained; by dividing *Unity* by the Number of cubic Inches contained in one Ale or Wine Gallon, or Malt Bushel.

Divisors for Squares, &c.	Factors for Squares, &c.	
282) 1.000000	(.003546	Ale Gallons.
231) 1.000000	(.004329	Wine Gallons.
2150.42) 1.00000000	(.000465	Malt Bushels.

Note. The above Factors, for multiplying the Square of the Diameter of any Circle (or the Product of the Transverse and Conjugate Diameters of any Ellipsis), may be otherwise obtained; *i. e.* by dividing *Unity* by the respective Divisors for Circles in Ale and Wine Gallons, and Malt Bushels.

The Gauge-points (on the Line D) on the Sliding-Rule, for Ale and Wine Gallons, and Malt Bushels, are the Square Roots of the Divisors for Squares, or Circles, in Ale and Wine Gallons and Malt Bushels, as follow.

Divisors for Squares, &c. $\left\{\begin{array}{c}282\\231\\2150.42\end{array}\right\}$, the Square Roots are

16.79

$\left.\begin{matrix} 16.79 \\ 15.19 \\ 46.37 \end{matrix}\right\}$ the Gauge-points for Squares.

Divifors for Circles, &c. $\left\{\begin{matrix} 359.053 \\ 294.118 \\ 2737.92 \end{matrix}\right\}$, the Square Roots are

$\left.\begin{matrix} 18.95 \\ 17.15 \\ 52.32 \end{matrix}\right\}$ the Gauge-points for Circles.

The above Gauge-points are manifeftly the Sides of Squares, and the Diameters of Circles, whofe Areas are one Ale or Wine Gallon, or Malt Bufhel.

A TABLE *of* Multipliers (or Factors), Divifors, *and* Gauge-points, *for* Squares *and* Circles, *&c.*

The Side of a Square, or the Diameter of a Circle, is 1. (Inch *&c.*)	Multipliers for Squares, *&c.*	Multipliers for Circles, *&c.*	Divifors for Squares, *&c.*	Divifors for Circles, *&c.*	Gauge-points for Squares, *&c.*	Gauge-points for Circles, *&c.*
	1	.785398	1	1.27324	1	1.128
Ale Gallon	.003546	.0027851	282	359.05	16.79	18.95
Wine Gallon	.004329	.00339999	231	294.12	15.19	17.15
Winchefter Malt (or Corn) Bufhel . .	.000465	.000365	2150.42	2738.00	46.37	52.32
Winchefter Malt (or Corn) Gallon . .	00372	.002922	268.8	342.24	16.39	18.5
A Pound of neat Tallow . . .	.031844	.025101	31.4	39.98	5.60	6.32
A Pound of hard Soap . . .	.036845	.028939	27.14	34.56	5.209	5.88
A Pound of green foft Soap . .	.038956	.0306	25.67	32.68	5.06	5.72
A Pound of white foft Soap . .	.0391235	.030731	25.56	32.54	5.05	5.7
A Pound of green Starch	.028735	.028563	34.8	44.32	5.9	6.65
A Pound of dry Starch	.024563	019493	40.3	51.3	6.34	7.16

PROP. 8.

Having given the Diameter of a Circle; to find its Area in Ale and Wine Gallons, and Malt Bushels, &c.

RULE.

Let the Square of the given Diameter be multiplied, or divided, by a Multiplier or a Divisor, agreeable to that Denomination in which the Area of the Circle is required; and the Product, or Quotient, will be the said Area sought.

EXAMPLE.

Suppose the Diameter of a Circle 68 Inches; required its Area in Ale and Wine Gallons, and Malt Bushels.

OPERATION.

$$
\begin{array}{r}
68 \\
68 \\
\hline
544 \\
408 \\
\hline
\end{array}
$$

The Square of the Diam. is 4624, which being multiplied (see the preceding Table) by

$$
\left\{\begin{array}{l}
.0027851 \text{ for Ale} \\
.0034 \text{ for Wine Gallons, and} \\
.000365 \text{ for Malt Bushels}
\end{array}\right\}, \text{ or divided by}
$$

$$
\left\{\begin{array}{l}
359.05 \text{ for Ale} \\
294.12 \text{ for Wine Gal. and} \\
2738.00 \text{ for Malt Bushels}
\end{array}\right\}, \text{ the Products, or}
$$

Quotients, give 12.87, 15.72, and 1.68, for the required Area of the Circle in Ale and Wine Gallons, and Malt Bushels respectively.

By

By the Sliding-Rule.

$$\text{To}\left\{\begin{array}{c}18.95\\17.15\\52.32\end{array}\right\}\text{marked}\left\{\begin{array}{c}\text{A.G}\\\text{W.G}\\\text{M.R}\end{array}\right\}\text{on D, ſet 1 on C;}$$

$$\text{then againſt 68 on D, is}\left\{\begin{array}{c}12.87\\15.72\\1.68\end{array}\right\}\text{on C; the ſame}$$

as before.

Prop. IX.

Having given the Length of the Arch, and the Semi-diameter (or Radius) of the Circle; to find the Area of the Sector in Ale and Wine Gallons, and Malt Buſhels.

Rule.

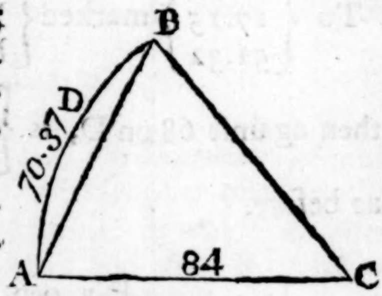
Multiply half the Length of the Arch by the Semi-diameter of the Circle; and divide the Product by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.

Example.

Let ADBC repreſent a Sector of a Circle, whoſe Semi-diameter AC (or BC) is 84 Inches, and the Arch ADB is 70.4 Inches; required the Area in Ale Gallons, &c.

Operation.

Operation.



½ the Arch ADB 35.185
Semi-diameter AC 84

140740
281480

Area in Inches 2955.540

282)2955.54(10.48 Ale Gallons
231)2955.54(12.8 Wine Gallons, *nearly*.
2150)2955.54(1.37 Malt Bushels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, set 84 on B; then opposite 35.18 on A, we have $\left\{\begin{array}{c}10.48\\12.8\\1.37\end{array}\right\}$ on B; the same as above.

But if the Arch ADB, or the Measure of the Angle ACB, in Degrees and Minutes is given, and likewise the Semi-diameter AC: Then multiply the Number of Degrees and Minutes (reduced to the Decimal Parts of a Degree) by the Square of the given Semi-diameter, and that Product by .000030945 for Ale, .000037777 for Wine Gallons, and by .000004059 for Malt Bushels.

In the preceding Example, the Arch ADB, or the Angle ACB, is found to be 48 Degrees, *nearly*; then 7056 (the Square of 84) being multiplied by 48, and that Product (which is 338688) by .000030945, gives 10.48 Ale Gallons: Moreover, the above Product (338688) being multiplied by

by .000037777 gives 12.8 Wine Gallons; and if 338688 be multiplied by .000004059, the Product will be 1.37 Malt Bushels; the very same as before.*

By the last Proposition, the Area of the Segment of a Circle may be obtained: — For if the Area of the Triangle ABC be subtracted from *that* of the Sector ADBC, there will remain the Area of the Segment ADBA: But since it is very troublesome to get the Length of the Arch of the Segment of a Circle; I shall therefore give one general Rule, whereby the Area of that Figure may be found to a very great Degree of Exactness, by having only its Chord and Versed Sine given; from whence the Diameter of the Circle is very easily obtained, by either of the following Methods.

If the Sum of the Squares of the Semi-chord and Versed Sine, be divided by the Versed Sine, the Quotient will be the Diameter of the Circle to which that Segment corresponds: — Or, if the Square of the Semi-chord be divided by the Versed Sine, and the Quotient thereof added to the Versed Sine, that Sum will likewise give the required Diameter.

Suppose

---

* If the Diameter of a Circle be $= 2$, its Circumference will be $=$ 6.2831, &c. and therefore $\frac{6.2831}{360}$ ($=$ .017453, &c.) will express the Length of the Arch of one Degree when the Radius of the Circle is 1: — Now let b denote any Number of Degrees and Minutes, &c. reduced to the Decimal Parts of a Degree; then will $.017453 \times b$ represent the Length of those Degrees, &c. in the same Measure of which the Radius is 1; then, because similar Arcs (as well as the whole Peripheries) of unequal Circles, are to one another as their Radii, we have 1 (the Radius of the less Circle) $: b \times .017453 :: s : bs \times .017453 =$ the Length of the Arch to the Radius s; $\therefore bs \times \frac{.017453}{2} \times s$, or $.00872664 \times bs^2 =$ the Content of the Sector in Inches; consequently the Content in Ale Gallons is $= \frac{.00872664}{282} \times bs^2$ $= .000030945 \times bs^2$.

Suppose, for Example, the Chord of a Segment of a Circle be 24, and its Versed Sine 8; required the Diameter of that Circle.

OPERATION.

```
                     Semi-chord 12
                                12
                          ----------
                               144
Add the Squ. of the V. Sine     64
                          ----------
                          8)208(26, the required
                            16       [Diameter.
                          ------
                             48
                             48
                          ------
                             ..
```

Otherwise, by the second Method.

The Square of the Semi-chord is 144, which being divided by the Versed Sine (8) gives 18, to which add the Versed Sine, and we have 26, the required Diameter, as above.

Both these Methods are very easily derived from the Properties of the Circle; which Properties are well known to Geometers.

PROP. X.

Having given the Chord and Versed Sine of the Segment of a Circle; to find its Area in Ale and Wine Gallons, and Malt Bushels.

RULE.

Rule.*

Divide the Difference between the Versed Sine and the Semi-diameter by 4, and *note* the Quotient.

1. Subtract the Square 2. Subtract four times the Square 3. Also take nine times the Square	of the above *noted* Quotient, from the Square of the Semi-diameter, and *note* the Difference.

Then to four times the Sum of the Square Roots of the first and third Differences, add twice the Square Root of the second; to this Sum add the Semi-diameter and Semi-chord.

Multiply that Total by $\frac{1}{6}$th Part of the Difference between the Semi-diameter and the Versed Sine; this Product being taken from 1.57079 times the Square of the Semi-diameter, leaves the Measure of the Segment in Inches; which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Bushels.

Example.

Required the Area of the Segment of a Circle, whose Chord is 40, and the Versed Sine 10 Inches.

* This Rule is very easily deduced, from the Method of equidistant Ordinates explained farther on.

Operation.

The Square of the Semi-chord is 400
The Square of the Versed Sine is 100

10) 500 (50 = the Diameter of the Circle.

$\frac{1}{4}$th of the Difference between the Semi-diameter (25) and the Versed Sine (10) } 3.75

The Square of 3.75 is 14.0625, which being multiplied by 4, gives 56.25; also 14.0625 being multiplied by 9, gives 126.5625: Then the Square of the Semi-diameter (25) being 625; therefore,

1. From 625 subtract 14.0625, the Remainder is 610.9375.

2. From 625 subtract 56.25, and there remains 568.75; and

3. From 625 take 126.5625, and there remains 498.4375.

The Square Root of the	1st	Difference is	24.7171
	2d		23.8484
	3d		22.3257

Four times the Sum of the 1st and 3d is	188.1712
Twice the 2d Square Root	47.6968
The Semi-diameter	25
The Semi-chord	20
Total	280.8680
Multiply by $\frac{1}{6}$th of the Diff. between the Semi-diameter and Versed Sine . . .	2.5
	14043400
	5617360
Product	702.17000

The

The Square of 25 (the Semi-diameter) multiplied by 1.57079 is	981.74
Subtract the above Product	702.17
The required Area of the Segment in Inches, *nearly*	279.57

282)279.57(.99 Area in Ale Gallons.
231)279.57(1.21 Wine Gallons.
2150)279.57(.13 Malt Bushels.

It may not be amiss to inform the Reader, that the greatest Error which can ever happen in computing, by the preceding Rule, the Area of any Segment of a Circle, will not exceed the *true* Measure $\frac{1}{600}$th Part of the Whole; and in many Circumstances, especially when the Segment approaches near to a Semi-circle, will be more exact than by any *general Rule* I have hitherto met with.

But the most expeditious Method of computing the Measure of the Segment of a Circle is, by a Table which is formed of the Areas of the Segments of a Circle,* whose Diameter is 1; and which

* It may not be improper to shew an easy Method of making this Table of the Areas of the Segments of a Circle; and likewise the Reason of finding, thereby, the Area of a Segment of any Circle, by having only the Diameter thereof, and the Versed Sine of the Segment given:—This last depends on the following

THEOREM.

From the common Center of any two concentric Circles, let two Right-lines be drawn to any two Points in the outermost Circumference, and also let the Chord of the Arc of each Circle, included by those two Lines, be drawn: Then will the Versed Sines of the Segments so formed, be to each other in the Ratio of the corresponding Radii, or Diameters, of the two Circles; and the Areas of those Segments will be as the Squares of those Radii, or Diameters.

Let OB and OD (Fig. IV.) be the two Right-lines drawn from the common Center O; draw the Chords *db* and DB, and perpendicular thereto draw the Radius OC; moreover let the Chords *cb* and CB be drawn. Then, by similar Triangles, we have, *cb* : CB :: O*b* : OB, and also *cb* : CB :: *cn* : C*m*; whence, by Equality, O*b* : OB :: *cn* : C*m*; or *cr* (2O*b*) : CE (2OB) :: *cn* : C*m*.

Moreover

which is supposed to be divided, by Chords perpendicular thereto, in 1000 equal Parts: For if the

---

Moreover, because similar Arcs of unequal Circles are as their corresponding Radii, it will be, as Ob : the Arc bec :: OB : the Arc BFC, and (by 15. *Eu.* 5.) as $Ob \times Ob$: the Arc $bec \times Ob$ (:: OB : Arc BFC) :: OB$\times$OB : the Arc BFC$\times$OB, that is Ob^2 : OB^2 :: the Sector $Odcb$: the Sector ODCB; but (by 19. *Euclid* 6.) Ob^2 : OB^2 :: the Triangle Odb : the Triangle ODB; whence, by Equality, the Sector $Odcb$: the Triangle Odb :: the Sector ODCB : the Triangle ODB; therefore, by Division, $Odcb-Odb$: Odb :: ODCB—ODB : ODB, or the Segment dcb : the Seg. DCB (:: Odb : ODB) :: Ob^2 : OB^2 (:: cr^2 : CE^2). Q. E. D.

A Method of computing the Table of the Areas of Segments of a Circle.

Suppose the Radius of a Circle = 1; then will the Measure of any Segment thereof, be expressed by $\frac{1}{2}$ the Measure of the Arc of that Segment, *minus* $\frac{1}{2}$ the Sine of *that* Arc: For it is evident, the former ($ceb \times 1$, Fig. IV.) expresses the Measure of the Sector $Odcb$, and the latter ($\frac{1}{2}bf \times 1$) the Measure of the Triangle Odb.

Now, in order to determine the Measure of the Segment of a Circle to any proposed Versed Sine, supposing the Radius = 1, and divided into 1000 equal Parts by perpendicular Chords: Take, out of a Table of natural Versed Sines, the Degrees, Minutes, and (by proportioning) the Seconds, answering to the Versed Sine proposed; this gives half the Angle at the Center, or half the Arch of the Segment: Then find the Measure thereof in Parts of the Radius (1); by multiplying the Number of Seconds therein, by the constant Factor .0000048481 expressing the Length of one Second; being = $\frac{6.28318}{1296000}$, *viz.* the whole Periphery of the Circle (to the Rad. 1.) divided by the Number of Seconds in 360°.

From the Measure of the Arc thus obtained, take $\frac{1}{2}$ the Sine of twice that Arc, the Remainder will express the Measure of the Segment to the Versed Sine proposed, when the Diameter of the Circle is supposed = 2: But if the Diameter of the Circle be = 1, which indeed is more commodious for Practice; then, by the preceding Theorem, the *Measure* of any Segment will only be $\frac{1}{4}$th of *that* of a similar Segment, when the Diameter of its Circle is supposed = 2. By this Means we derive the very same Table, as that given at the End of *Shirtcliffe*'s Gauging.

Suppose, for Example, it was proposed to find the Measure of the Segment of a Circle whose Diameter is 2, and the Versed Sine .1.—Let the Radius (1) be conceived to be divided into 1000 equal Parts, then the proposed Versed Sine will be represented by 100; for, by the preceding *Theorem*, 1 : .1 :: 1000 : 100; then in *Sherwin*'s Tab. of nat. Versed Sines, against 999.346 we have 25° 50′, and also against 1000.614 we have 25° 51′; ∵ 1.268 (*viz.* 1000.614—999.346) : 60″ :: .654 (1000—999.346) : $\frac{.654 \times 60}{1.268}$ = 31″ *very nearly*; then will 25° 50′ 31″ express half the Arc of the Sector (or Segment), the double whereof is 51° 41′ 2″, the Sine of which is .7845961; but 25° 50′ 31″ = 93031″, which being multiplied by .0000048481,

the Versed Sine and Diameter of a Circle are known, (or the Versed Sine and the Chord of a Segment of a Circle from whence the Diameter becomes known, see *Page* 88); then will the Measure of the Segment be obtained by the following easy

RULE.

Divide the Versed Sine of the Segment (with a competent Number of Cyphers annexed) by the Diameter of its Circle, to three Places of Decimals in the Quotient; find this *Quotient* in the Table of Areas of the Segment of a Circle, under the Letters V. S. and then against it, under *Seg. Area*, is a Decimal Number; which being multiplied by the Square of the given Diameter, the Product will be the required Measure of the Segment.

EXAMPLE.

Suppose the Diameter of a Circle to be 80 Inches; required the Area of a Segment thereof (in Ale Gallons, *&c.*) whose Versed Sine is 30 Inches.

OPERATION.

.0000048481, the Length of the Arc of 1″ (to the Radius 1), gives .45102359, for the Measure of the Arc 25° 30′ 31″ in Parts of the Radius 1.

Subtract ½ the Sine of the whole Arc, *viz.* Half .7845961 . . . . } .39229805

Remains the required Area, when the Diameter is = 2 . . . . } .05872554

¼th of which (see the preceding *Theo.*) is .01468138, the Area of the Segment of a Circle whose Diameter is 1, and Versed Sine .1 or .05; *viz.* 100 or 50, according as the Radius is supposed to be divided into 1000, or 500 equal Parts.

OPERATION.

80) 30.000 (.375 *Quotient*
240
———
600
560
———
400
400
———
. .

Under the Letters V. S, find the above Quotient .375, againſt which is .269013, this being multiplied by 6400 the Square of the Diameter, the Product is 1721.6832, the Area of the Segment in Inches; which being divided by 282 gives 6.105 Ale Gallons, and being divided by 231, the Quotient will be 7.45 Wine Gallons.

If the Area of the above Segment be computed by the foregoing *general Rule*, the Reſult will be 6.105 Ale, and 7.45 Wine Gallons; *exactly as above.*

PROP. XI.

The Tranſverſe and Conjugate Diameters of an Ellipſis being given; to determine the Area thereof in Ale and Wine Gallons, and Malt Buſhels.

RULE.

Multiply the Tranſverſe (or longeſt) Diameter, by the Conjugate (or ſhorteſt) Diameter; and let that Product be multiplied, or divided, by a Multiplier, or a Diviſor, agreeable to that Denomination in which the Area of the Ellipſis is required; and the Product, or Quotient, will be the Anſwer ſought.

EXAMPLE.

Example.

Suppose the Transverse Diameter of an Ellipsis to be 70, and the Conjugate 50 Inches; required its Area in Ale Gallons, &c.

Operation.

70
50

The Product of the two Diameters } 3500, which being multiplied (see the *Table Page* 83) by
.0027851 for Ale
.0034 for Wine Gallons, and
.000365 for Malt Bushels; }, or divided by
359.05 for Ale
294.12 for Wine Gallons, and
2738.00 for Malt Bushels }; the Products, or Quotients, give 9.74, 11.90, and 1.27 for the required Area of the Ellipsis in Ale and Wine Gallons, and Malt Bushels respectively.

By the Sliding-Rule.

To { 359, 294, 2738 } on A, set 50 on B; then against 70 on A, we shall have { 9.74, 11.90, 1.27 } on the Line B; the Areas as before.

Prop. XII.

Having given the Base and Perpendicular of a Parabola, (or the Ordinate and Abscissa, see Defin. 9. Page 63); *to determine the Area thereof in Ale and Wine Gallons, and Malt Bushels.*

Rule.

RULE.

Multiply the Base (or Ordinate) by $\frac{2}{3}$ds of the Perpendicular (or Abscissa); and divide the Product by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Bushels.

EXAMPLE.

Let the Base (or Ordinate) of a Parabola be 64, and the Perpendicular (or Abscissa) 36 Inches; required the Area in Ale Gallons, *&c.*

OPERATION.

$$\begin{array}{rr} & 64 \\ \frac{2}{3}\text{ds of } 36 \text{ (the Perp.) is} & 24 \\ \hline & 256 \\ & 128 \\ \hline \end{array}$$

282)1536(5.44, the Area in Ale Gallons.

231)1536.00(6.64 Wine Gallons.

2150)1536.00)0.71 Malt Bushel.

By the Sliding-Rule.

To $\left\{\begin{array}{c} 282 \\ 231 \\ 2150 \end{array}\right\}$ on A, set 64 (or 24) on B; then opposite 24 (or 64) on A, we shall have $\left\{\begin{array}{c} 5.44 \\ 6.64 \\ 0.71 \end{array}\right\}$ on the Line B; the Areas as above.

PROP. XIII.

Having the Transverse and Conjugate Diameters of an Ellipsis given; to find the Area of any Segment thereof, (formed by drawing a Line parallel to either of those Diameters.)

RULE.

RULE.

Find (by *Prop*. 10. *Page* 89.) the Area of a Circular Segment, whose Versed Sine is the Altitude of the Elliptic Segment, and the Diameter of the Circle is the Transverse (or Conjugate) Diameter of the Ellipsis; Then, if the Elliptic Segment is formed by a Line parallel to the Conjugate Diameter, multiply the Area of the Circular Segment by the Conjugate, and divide the Product by the Transverse Diameter: But if the Elliptic Segment is made by a Line drawn parallel to the Transverse Diameter; then multiply the Area of the said Circular Segment by the Transverse Diameter, and divide the Product by the Conjugate,* the Quotient (in each Case) will be the Area of the Elliptic Segment (in Inches, *&c.*); which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Bushels.

---

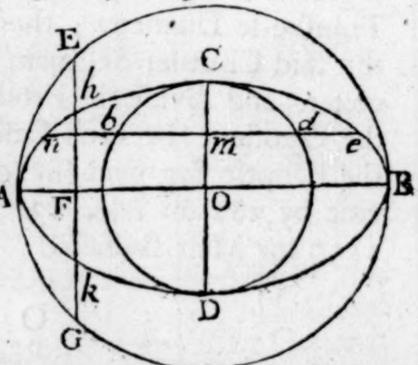
* Let the Transverse Axis AB $=a$, the Conjugate CD $=c$, the Abscissa C$m=x$, and the Ordinate $mn=y$ (see the following *Fig.*): Then, for the very same Reason that $\frac{c}{a}\times\dot{x}\sqrt{ax-x^2}$ (or $y\dot{x}$) is the Fluxion of the Elliptic Segment kAb (when AF $=x$ and F$b=y$), will $\frac{a}{c}\times\dot{x}\sqrt{cx-x^2}$ be the Fluxion of the Elliptic Segment nCe; but $\dot{x}\sqrt{cx-x^2}$ is the Fluxion of the circular Segment bCd (see *Simpson*'s Fluxions, Page 146); let the Fluent thereof be $=$ A; then the Fluent of $\frac{c}{c}\times\dot{x}\sqrt{cx-x^2}$ is $\frac{a}{c}\times$ A; whence the Area of the *Circular Segment* bCd is to that of the *Elliptic Segment* nCD, as A : $\frac{a}{c}\times$ A (or 1 : $\frac{a}{c}$) or c : a; that is, as CD : AB :: bCd : nCe.

EXAMPLE.

In the Ellipsis ADBC, let the Transverse Diameter AB be 82, and the Conjugate CD be 52 Inches, and suppose the Versed Sine AF to be 8.5 Inches; to find the Area of the Elliptic Segment *kAb* in Ale Gallons, *&c.*

OPERATION.

82) 8.500 (.103



Against the Versed Sine .103, in the Table for the Segments of Circles, is 042687, which multiply by the Square of 82 *viz.* 6724, gives 287.027, the Area (in Inches) of the Circular Segment GAE; this Area being multiplied by 52, and the Product thereof divided by 82 (agreeable to the former Part of the preceding Rule) the Quotient will come out 182.01 Inches, the Area of the Elliptic Segment *kAb*: Whence the Area of the said Segment, in Ale and Wine Gallons, and Malt Bushels, is easily found; by dividing 182.01 by the above Divisors respectively.

The Method of Operation, for finding the Area of the Elliptic Segment *nCe*, is the same as above; only observe, that the Area of the Circular Segment (*bCd*) is to be multiplied by the Transverse (instead of the Conjugate) Diameter, and that Product divided by the Conjugate (instead of the Transverse) Diameter.

PROP.

Prop. XIV.

To find the Side of a Square inscribed in a Circle, whose Diameter is given.

Rule.

Multiply the given Diameter of the Circle by .707, and the Product will be the Side of the required Square, *nearly*.*

Example.

Suppose the Diameter of a Circle be 62.5 Inches; what is the Side of the inscribed *Square*; or the *greatest* that can be formed within that Circle?

Operation.

$$\begin{array}{r} 62.5 \\ .707 \\ \hline 4375 \\ 43750 \\ \hline \end{array}$$

Product is 44.1875, the Side of the Square.

By the Sliding-Rule.

To Unity on A, set .707 (marked *s. i.*) on B; then against 62.5 on A, is 44.2 on B.

 N. B.

* In every Circle, the Chord of 90° is manifestly the Side of the inscribed Square; and therefore, when the Diameter of the Circle is Unity, the Side of its inscribed Square will (by 47. *Eu.* 1.) be expressed by $\sqrt{\frac{1}{2}}$, or .707 &c. whence, by similar Triangles, it will be, as 1 : .707 :: *a* (any given Diameter) : *a* × .707, the Side of the Square inscribed in a Circle, whose Diameter is *a*, *nearly*. Q. E. I.

N. B. This Propoſition is very uſeful in the quartering of a round Tun, *&c.* as will be ſhewn further on.

It may be proper to obſerve, that when the Content of any Veſſel is known in cubic Inches, its Content in Pounds of Glaſs may readily be obtained, by dividing the ſaid cubic Inches by the proper Diviſor, as follows.

			Diviſors.	
A Pound Avoirdupoiſe Weight of	*Flint Glaſs*	contains	8.46	cubic Inches.
	Plate Glaſs		9.178	
	Crown and *Broad Glaſs*		10.516	
	Phial and *Bottle Glaſs*		10.178	

Vid. the *Officers Inſtructions* for charging the Duties on Glaſs.

Hence the correſponding circular Diviſors, Factors, and Gauge-points, may be eaſily obtained by the Methods laid down *Pa.* 82.

SECTION

SECTION VIII.

OF THE MEASURE OF SOLID FIGURES; *or of finding their Contents in Ale and Wine Gallons, and Malt Bushels.*

THE Measure of every solid Figure is computed from another Solid, of a determinate Form and Magnitude; namely, from a *Cube*, whose Side is one *Inch*, *Foot*, *Yard*, *&c.* called the *measuring Unit*, and the Number of such *Cubes*, or *Units*, (and Parts of an Unit) that any Solid is found to contain, is called the *Measure*, or *Content*, of the Solid: Therefore when the Measure of any Figure in cubic Inches is known, its Measure in Ale and Wine Gallons, and Malt Bushels will be easily found, by dividing the said cubic Inches by the proper Divisors for those Measures respectively.

PROP. I.

The Side of a Cube being given in Inches; to find its Content in Ale and Wine Gallons, and Malt Bushels.

RULE.

The Length, Breadth, and Altitude of the Cube (which are all equal) being multiplied together, gives the Content in cubic Inches; which divide

divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Bushels.

EXAMPLE.

Suppose the Side of a Cube to be 15 Inches; required its Content in Ale Gallons, &c.

OPERATION.

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \end{array}$$

The Content of the Cube in Inches 3375

282)3375.00(11.96 Ale Gallons.
231)3375.00)14.61 Wine Gallons.
2150)3375.00)1.57 Malt Bushel.

By the Sliding-Rule.

To $\left\{\begin{array}{l} 16.76 \\ 15.19 \\ 46.37 \end{array}\right\}$ on D, set 15 on C; then opposite 15 on D, is $\left\{\begin{array}{l} 11.96 \\ 14.61 \\ 1.57 \end{array}\right\}$ on C; the same as before.

PROP.

PROP. II.

The Length, Breadth, and Depth (or Altitude) of a rectangular Parallelopipedon being given in Inches; to find its Content in Ale and Wine Gallons, and Malt Bushels. (See Definition 25, *Pa.* 58.)

RULE.

Multiply the Length by the Breadth, and that Product by the Depth (or Altitude), the last Product will be the Content in cubic Inches; which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Bushels.

EXAMPLE.

The Length of a rectangular Parallelopipedon is 72, the Breadth 33, and the Depth (or Altitude) 82 Inches; required the Content in Ale and Wine Gallons, and Malt Bushels.

OPERATION.

Length	72
Breadth	33
	216
	216
The Area of the Base	2376
	82
	4752
	19008
Content in cubic Inches	194832

282)194832.00(690.89 the Content in Ale [Gallons.

231)194832.00(843.42 Wine Gallons.

2150)194832.00(90.61 Malt Bushels.

By the Sliding-Rule.

$$\text{To}\left\{\begin{matrix}282\\231\\2150\end{matrix}\right\}\text{on A, ſet 33 (or 72) on B; then}$$

Areas.

$$\text{againſt 72 (or 33) on A, is}\left\{\begin{matrix}8.4\\10.2\\1.11\end{matrix}\right\}\text{on B.}$$

$$\text{Then to}\left\{\begin{matrix}8.42\\10.2\\1.11\end{matrix}\right\}\text{on A, ſet 1 on B; and oppo-}$$

$$\text{ſite 82 on B, is}\left\{\begin{matrix}690.8\\843.4\\90.6\end{matrix}\right\}\text{on A.}$$

Otherwiſe by the Sliding-Rule.

A Mean-Proportional between the Length (72) and the Breadth (33) is 48.74 (found by *Prop.* 3. *Page.* 46): Then,

$$\text{To}\left\{\begin{matrix}16.79\\15.19\\46.37\end{matrix}\right\}\text{on D, ſet 82 on C; and againſt}$$

$$\text{48.74 on D, is}\left\{\begin{matrix}690.89\\843.42\\90.61\end{matrix}\right\}\text{on C; the ſame as above.}$$

The foregoing Methods are both very exact and expeditious, for computing the Contents of many Brewers *Guile-Tuns* and Diſtillers *Waſh-Backs*; that is, ſuch whoſe four Sides ſtand perpendicular to the Bottom, which (in this Caſe) is a rectangular Parallelogram: But, it is to be obſerved, that, on Account of the Unevenneſs of the Sides of the Tun *&c.* it will be very neceſſary to take 10 Lengths (at leaſt), and as many Breadths; each, as

as near as possible, at equal Distances from one another, and from the Sides of the Vessel; then the Sum of the Lengths being divided by 10 (or the Number of Lengths taken) and the Sum of the Breadths divided in like Manner; or, which comes to the same Thing, the Decimal Point, in each of these Sums (*viz.* when there are 10 Lengths and 10 Breadths), being removed one Place more towards the Left-Hand, gives the mean Length and Breadth of the Tun, or Back: Suppose, in the last Example, the Lengths and Breadths were taken, each at 10 different Places, as follow:

	Lengths.	Breadths.
	71.8	32.7
	72.4	33.4
	72.2	33.1
	71.7	33.0
	72.0	32.7
	71.8	32.9
	71.8	33.2
	72.0	33.1
	72.2	33.3
	72.1	32.6
Total	720.0	330.0
$\frac{1}{10}$th is the mean Length . .	72.00	$\frac{1}{10}$th is 33.00 the mean Breadth.

At common Brewers, *&c.* the Coolers (or Backs) are generally in the Form as described above; but, as their Depths seldom exceed 8 Inches, it will be sufficient to take (about the Middle) only one Length and one Breadth; which being multiplied together, and the Product thereof divided by 282, the Quotient will be the Area of the Cooler in Ale Gallons.

Example.

Suppose the Length of a Cooler to be 112.5, and the Breadth 82.2 Inches; required its Area in Ale Gallons.

Operation.

```
     112.5
      82.2
    ------
      2250
     2250
    9000
    ------
282)9247.50(32.8 Area in Ale Gallons.
```

The Wort in the Coolers being always gauged to the tenth of an Inch, it is therefore necessary that the Tables of such Vessels should be made in Barrels, Firkins, &c. to every Tenth; which is usually called *tenthing* a Cooler: But, before we enter upon that, it will be proper to observe, that, at common Brewers, there is an Allowance made of one Gallon in ten, on Account of the Heat of the Wort; that is, every 10 Gallons of hot Wort in the Coolers, will be but 9 Gallons when cold, and let down into Tun; consequently a Table must be made of only $\frac{9}{10}$ths of the whole Area of the Cooler, as follows.

Whole Area 32.8 Gallons.
Subtract $\frac{1}{10}$th 3.28

Remains 29.52, the neat Area for one

Inch, $\frac{1}{10}$th of which is 2.952, the *neat Area* of the Cooler for $\frac{1}{10}$th of an Inch; but the 3d Decimal Figure (being in this Case of small Value) may be rejected, and therefore 2.95 will be the neat Area;

Area; by the continual Addition of which, the following Table of *Beer Barrels* was made.

Note. The neat Area of any Back (or Cooler), for $\frac{1}{10}$th of an Inch, may alſo be found by multiplying the whole Area thereof (*viz.* for one Inch deep) by .09: — Thus, for Example, the foregoing Area 32.8 being multiplied by .09, gives 2.952 the neat Quantity for $\frac{1}{10}$th; the ſame as before.

Tenths.	*B.*	*F.*	*Gall.*	*Parts.*
.1	0	0	2	95
.2	0	0	5	90
.3	0	0	8	85
.4	0	1	2	80
.5	0	1	5	75
.6	0	1	8	70
.7	0	2	2	65
.8	0	2	5	60
.9	0	2	8	55
1.0	0	3	2	50
&c.		*&c.*		

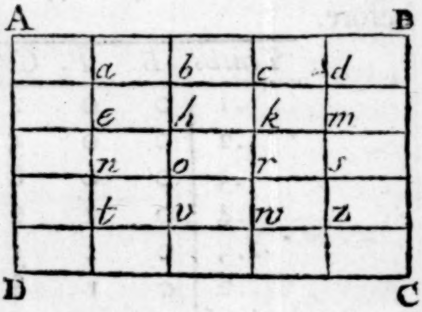
I thought it needleſs to proceed any farther with the foregoing Table, ſeeing that the Method of forming it is only the continual Addition of 2.95 Gallons.

By the ſame Method the Back may be tabulated for *Ale Barrels*, *&c.* (*i. e.* 34 Gallons to a Barrel), due Regard being had to the Decimal Parts, when the Sum of the Gallons and Parts of the two Numbers to be added, exceeds one Firkin (*i. e.* 8.5 Gallons).

It is, indeed, wholly immaterial in what Part of a Cooler the Gauge of the Wort is taken, provided its *Bottom* is fixed in an horizontal Poſition: But it is well known, that *that* is always placed a little inclined, for the Convenience of the Wort's running out: Beſides, at common Brewers, large Backs are generally found to ſettle of themſelves,

more one Way than another; and moreover their Bottoms will frequently warp, and thereby cauſe ſuch an Unevenneſs in them, as to render it almoſt impoſſible to know where to take a Dip of the Wort, whereby the true Quantity may be aſcertained.

Now in Order to find, with the moſt Certainty, a mean Dip of a rectangular Back or Cooler, proceed thus: Let its Length and Breadth be each divided at the Bottom, into 4, 5, 6, 7, *&c.* equal Parts, according to the Magnitude of the Back, and the Irregularity of its Bottom, alſo



let parallel Chalk-lines be ſtruck; ſee the above Figure ABCD; which may be ſuppoſed to repreſent the Bottom of a rectangular Cooler: Then (the Bottom being covered with Water) let Dips be taken at all the Points of Interſection (*a, e, n, t, v, &c.*) of thoſe parallel Lines; the Sum of which Dips being divided by the Number of Dips taken, will give the mean Depth (or Dip) ſought.

Find in what *Place* of the Back, a Dip being taken, will anſwer the mean Dip, for *that* muſt be *noted* for the conſtant Dipping-place: But if ſuch Place cannot eaſily be come at, then chooſe One which will be the moſt convenient to dip at, and there make ſome immoveable Mark; obſerve how much the Dip taken at this Place falls ſhort, or exceeds the mean Dip (found as above), and accordingly mark it down on the Side of the Back, at the fixed Dipping-place, with the Character + or —: Suppoſe, for Example, the mean Dip of a Cooler be 4.5; and, at the intended Dipping-place, it is found

to

to dip only 4 Inches; therefore it is plain that .5 muſt be added to every Gauge (or Dip) that is taken of the Wort, at the fixed Dipping-place, and muſt be there marked thus, + 0.5: But if the Dip at this Place, had been 5 Inches, (which exceeds the mean Dip by .5), we muſt then have marked the Dipping-place —0.5.

Note. If the Sides of a Back *&c.* are parallel, and there happens to be any conſiderable Difference between the two Diagonals: — Then we muſt mark (with a Chalk-line on the Bottom) the longeſt Diagonal, and let Perpendiculars fall thereon, from the two oppoſite Angles, as in the Trapezium *Pa.* 74.

It will be unneceſſary to give Examples for finding the Contents of all the various Sorts of Priſms which may occur in Practice, if the 25th *Definition*, *Pa.* 58, be rightly underſtood; for the Method of Operation, by the Pen, is much the ſame as *that* of the foregoing Examples, let the Figure of the two equal Ends of the Priſm be what it will: — That is, *multiply the Area of one of the Ends, by their perpendicular Diſtance aſunder, and the Product will be the Meaſure of the Priſm in cubic Inches; which divide by* 282 *for Ale,* 231 *for Wine Gallons, and* 2150 *for Malt Buſhels.*

Note. The preceding Method of dividing the Bottom of a Cooler, will be very uſeful in gauging of a large Veſſel in the Form of a rectangular Parallelopipedon (ſee *Pa.* 103).—For, if the Veſſel is of a conſiderable Depth and Magnitude, the Sides (tho' intended by the *Artiſt* to ſtand perpendicular to the Bottom) are very ſubject to warp, bulge, and have many Irregularities in them. The beſt Method, therefore, of obtaining a mean Length and mean Breadth, will be to divide and ſtrike Chalk-lines on each of the four Sides of the Veſſel, in the very ſame Manner as was deſcribed

ſcribed for the Bottom of a rectangular Cooler, (ſee the preceding *Fig.)*; and then to take all the Lengths and Breadths at the oppoſite Interſections of thoſe Lines, in Order to get a mean Length and Breadth. (See *Pa.* 105).

Prop. III.

The Diameter and Length of a Cylinder being given; to find its Content in Ale and Wine Gallons, and Malt Buſhels.

Rule.

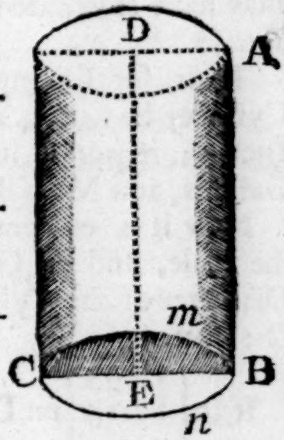
Multiply the Square of the Diameter by the Length, or Altitude, of the Cylinder; and divide the Product by 359 for Ale, 294 for Wine Gallons, and 2738 for Malt Buſhels: — Or the ſaid Product being multiplied (ſee *Ta. Pa.* 83) by .0027851, .0034 and .000365, will give the Content of the Cylinder in Ale and Wine Gallons, and Malt Buſhels reſpectively.

Example.

The Diameter of a Cylinder BC is 32, and the Altitude, or Length, AB 45.5 Inches; required its Content in Ale Gallons, &c.

Operation.

OPERATION.



32
32
———
64
96
———
The Square of the Diam. 1024
Length 45.5
———
5120
5120
4096
———
359)46592.00(129.78 the Content in Ale Gallons.

294)46592.00(158.47 Wine Gallons.

2738)46592.00)17.01 Malt Bushels.

By the Sliding-Rule.

To $\left\{\begin{matrix}18.95\\17.15\\52.32\end{matrix}\right\}$ marked $\left\{\begin{matrix}A.G\\W.G\\M.R\end{matrix}\right\}$ on D, set 45.5 on C; then against 32 on D, is $\left\{\begin{matrix}129.78\\158.47\\17.01\end{matrix}\right\}$ on C; the same as before

If the Diameter be less than 10, or more than 100; or if it so happens, that, when the Length of the Cylinder on C is set to any of the aforesaid Gauge-points on D, the Diameter of the Cylinder on D, should fall off the Slide either towards the Right

Right or Left-Hand: Then, in Order to find the Content of the Cylinder by the *Sliding-Rule*, we may have Recourse to the Method laid down in *Pa.* 48.

Thus, for Example, suppose the Length of a Cylinder be 45.5, and the Diameter thereof 8 Inches; required its Content in Ale and Wine Gallons, and Malt Bushels.

Now it is evident, from the Construction of the Rule, and the Gauge-points thereon, that the Diameter of the Cylinder (8) will fall off the Line D: But,

If to $\left\{\begin{array}{l}18.95\\17.15\\52.32\end{array}\right\}$ on D, be set 45.5 on C; then against 16 (the Double of 8) on D, we shall have $\left\{\begin{array}{l}32.5\\39.6\\4.25\end{array}\right\}$ on C; which being divided by 4 (because the Diameter of the Cylinder was doubled), gives $\left\{\begin{array}{l}\text{8.12 Ale Gallons}\\\text{9.9 Wine Gallons}\\\text{1.06 Malt Bushels}\end{array}\right\}$ the required Content of the Cylinder.

PROP. IV.

To find the Content of a Pyramid (or Cone) in Ale and Wine Gallons, and Malt Bushels. (See Definitions 1 and 27, *Pages* 59 and 61.)

RULE.

Multiply the Area of the Base (let the Figure thereof be what it will) by $\frac{1}{3}$d of the Altitude, and the Product will be the Content in cubic Inches; which being divided by 282, 231, and 2150,

2150, the Quotient will be the required Content in Ale and Wine Gallons, and Malt Bushels respectively.

EXAMPLE.

Suppose the Side of the Base of a square Pyramid be 35, and the Altitude 57 Inches; required its Content in Ale Gallons, &c.

OPERATION.

	35
	35
	175
	105
The Area of the Base in Inches	1225
$\frac{1}{3}$d of the Altitude	19
	11025
	1225
The Content of the Pyramid in Inches	23275

282)23275.00(82.53, the Content in Ale Gallons.

231)23275.00(100.75 Wine Gallons.

2150)23275.00(10.82 Malt Bushels.

By the Sliding-Rule.

To $\begin{Bmatrix} 16.79 \\ 15.19 \\ 46.37 \end{Bmatrix}$ on D, set 19 on C; then against 35 on D, is $\begin{Bmatrix} 82.53 \\ 100.75 \\ 10.82 \end{Bmatrix}$ on C; the same as before.

Q PROP.

PROP. V.

To find the Content of the Frustum of a Cone (or Pyramid of any Kind) in Ale and Wine Gallons, and Malt Bushels. (See Defin. 32. *Pa.* 60).

RULE.

To the Sum of the Areas of the two Ends of the Frustum, add a Geometrical-Mean between those Areas (*i. e.* the Square Root of their Product), multiply this Sum by $\frac{1}{3}$d of the Altitude of the Frustum, the Product will give the Content thereof in cubic Inches;* which being divided as in the last Example, gives the Content sought.

The preceding Rule is general, let the Figure of the two (similar) Ends of the Frustum be what it will; but the Content of the Frustum of a Cone in Ale Gallons, &c. is more expeditiously obtained by the following

---

* Let the *Measure* of the greater End of the Frustum of any Cone, or Pyramid whatever (see *Fig.* V.), be denoted by A^2, *that* of the less End by a^2; then the Diameters, or any two homologous Sides of those Ends (because of their Similarity), will be as A and a respectively: Moreover let the Distance (*bm*) of the Ends be denoted by p, and the Perpendicular F*n* by x; and conceive the Solid (A*acb*BCA) to be cut by a Plane parallel to one of its Sides, so as to form a Pyramid (or a Prismoid) HI*b*B, and parallel to that Plane, let another be supposed to pass from any Altitude x: Then it is manifest, that p (*lm*) : $A-a$ (which is as BH) :: x (F*n*) : $\frac{A-a}{p} \times x$ (which is as B*r*); ∵ $A - \frac{Ax-ax}{p}$ (being as AB—B*r*) will be as EF; consequently the Area of the Section E G F (parallel to the Ends of the Solid) will be expressed by $\overline{\frac{Ap-Ax+ax}{p}}\Big|^2$; whence the Fluxion

of

RULE.

From the Square of the Sum of the top and bottom Diameters, ſubtract the Product of thoſe Diameters; the Remainder being multiplied by the Altitude of the Fruſtum, and the Product divided by 1077 for Ale, 882.36 for Wine, and 8214 for Malt Buſhels, gives the required Content.†

---

of the Solid, univerſally, will be $\overline{\frac{Ap - Ax + ax}{p}}\Big|^2 \times \dot{x}$, or (when expanded) $\frac{A^2p^2\dot{x} - 2A^2px\dot{x} + 2Apax\dot{x} + A^2x^2\dot{x} - 2Aax^2\dot{x} + a^2x^2\dot{x}}{p^2}$, whoſe Fluent is $\frac{A^2p^2x - A^2px^2 + Apax^2 + \frac{A^2x^3}{3} - \frac{2Aax^3}{3} + \frac{a^2x^3}{3}}{p^2}$; which, when $x = p$, becomes $Apa + \frac{A^2p}{3} - \frac{2Apa}{3} + \frac{pa^2}{3}$, or $\overline{A^2 + Aa + a^2} \times \frac{p}{3}$. Q. E. I.

† Suppoſe B and b denote the Diameters of two Circles, whoſe Areas are A^2 and a^2 reſpectively; that is, let $B^2 \times .7854 = A^2$, and $b^2 \times .7854 = a^2$: Then will $Bb \times .7854 = Aa$, and $\because \overline{B^2 + Bb + b^2} \times .7854 = A^2 + Aa + a^2$; whence $\overline{B^2 + Bb + b^2} \times .7854 \times \frac{p}{3}$ $(= \overline{B + b}|^2 - Bb \times .7854 \times \frac{p}{3})$ $= \overline{A^2 + Aa + a^2} \times \frac{p}{3} =$ the Content of a Fruſtum of any pyramidic Solid whoſe Altitude is p: See the preceding *Note*.

Hence it is very eaſy to deduce another general Rule for determining the Content of the Fruſtum of a Cone: For $\overline{\overline{B - b}|^2 + 3Bb} \times .7854 \times \frac{p}{3}$ $(= \overline{B^2 + Bb + b^2} \times .7854 \times \frac{p}{3})$ is likewiſe $= \overline{A^2 + Aa + a^2} \times \frac{p}{3}$.

EXAMPLE.

Let the top Diameter be 22, the bottom Diameter 40, and the Altitude (or Depth) 60 Inches; required the Content of the Frustum in Ale Gallons, &c.

OPERATION.

	22	
	40	
	62	22
	62	40
	124	880, the Product
	372	of the Diameters.
Sq. of the Sum of the Diam.	3844	
Subtract	880	
Remainder	2964	
Depth	60	

1077)177840(165.12, the Content of the Frustum in Ale Gallons; whence, by the proper Divisors, the Content in Wine Gallons and Malt Bushels, will be found to be 201.55 and 21.63 respectively.

Provided a Brewer's Guile-Tun (or a Distiller's Wash-Back, &c.) was a perfect Frustum of a Cone, the above Rule would be sufficient for finding its Content; and moreover a general Method might be given for finding the *true* Quantity upon every Inch of the Frustum's Altitude, by which Means a Table might be formed to know the Quantity of Liquor contained in the Tun (or Back),

Back), at any Number of wet (or dry) Inches of its whole Depth.

But, in Veſſels of this Kind, it is well known that the Croſs-Diameters, in various Parts of the Altitude, differ about an Inch, or an Inch and a Half, more or leſs, according as the Veſſel is of a greater or leſs Magnitude: Therefore the moſt practical and certain Method of finding the Content, and tabulating a Guile-Tun, &c. reſembling the Fruſtum of a Cone is, to take Croſs-Diameters in the Middle of every 6, 7, 8, 9, or 10, &c. Inches of its Altitude; then will half the Sum of any two Croſs-Diameters be, *nearly*, the true Diameter at that particular Altitude.

Find the Areas in Ale or Wine Gallons correſponding to the Diameters, thus obtained, in the Middle of every 6, 7, 8, 9, or 10, &c. Inches; the Sum of theſe Areas being multiplied by their common Diſtance aſunder; or, which is the ſame Thing, each Area being multiplied by its correſponding Part of the Depth, the Sum of the Products will give the Content of the Tun in Gallons, if it ſtands perpendicular to the Horizon: But if the Tun ſtands inclined; then ſo much Liquor muſt be meaſured therein, as will be ſufficient to cover the Bottom, and at the Place where the Depth of the Tun was found (which is always taken at the intended Dipping-place), take the Depth of the Liquor which covers the Bottom; that being ſubtracted from the whole Depth, leaves the *neat Depth* of the Tun.

If the Difference of the Croſs-Diameters of a Veſſel is pretty conſiderable (*i. e.* 4, 5 or 6 &c. Inches, more or leſs, according to its Magnitude); then every practical Gauger, who is more ſolicitous for Truth than Expedition, will have Recourſe to the *general Method* of equidiſtant Ordinates (Sect. XII.): There being no other Method,

thod, that I know of, for determining, with ſuch an amazing Degree of Certainty, the Meaſure of a curvilineal Figure, whoſe true Form (or Property) is unknown.

Now to quarter a Tun, or to obtain Croſs-Diameters of any Veſſel of a circular Form, in any Part of its Altitude, proceed thus.

With a Chalk-Line and Plummet, at the loweſt Part of the Tun, ſtrike a ſtraight Line on the Side thereof, from the Bottom to the Top; then with a Dimenſion-Rod take the Diameter of the Tun at the Bottom, multiply that Diameter by .707, (ſee *Pa.* 99.), the Product will be the Side of the inſcribed Square, *very nearly*, with which the Tun may be quartered at the Bottom: And by the very ſame Method of proceeding, the Tun (or Back) may be quartered at the Top; then let Marks be made with Chalk at each Quarter, both at the Bottom and Top, and ſtrike ſtraight Chalk-lines from thoſe Marks; the Veſſel will then be properly quartered, and Croſs-Diameters may be taken at any aſſigned Diſtance from the Bottom.

It may be proper to obſerve, that by Means of quartering a round Tun (or any inclined circular Veſſel) both at the Top and Bottom, we get the *true* Croſs-Diameters at any Part of its Depth; which otherwiſe could not be obtained, unleſs the Veſſel was fixed perfectly upright, or with its Bottom parallel to the Horizon.

It is moreover to be obſerved, that, when great Exactneſs is required, the Number of Areas, in every Veſſel (whether ſtraight or curve-ſided) ſhould be ſuch, that the Increaſe of the Croſs-Diameters (or Dimenſions) may not exceed one Inch: — In Order to obtain which, in a ſtraight-ſided Veſſel; divide the Altitude thereof by the Difference of the top and bottom Diameters (or Dimenſions), and the Quotient will be the perpendicular

pendicular Distance which the Cross-Diameters, *&c.* are to be taken from each other.*

If a Vessel is to be tabulated for the dry Inches, it will be proper to begin from the *Top*, to mark out and take its Dimensions; and from the *Bottom*, when it is to be tabulated for wet Inches.

When the Difference of the top and bottom Diameters (or Dimensions) of any straight-sided Vessel is but small; then the Distance of the Cross-Diameters, *&c.* may be set off upon the Side, without sensible Error: But, when that Difference is large, we must, in Order to have the *true* Distance of the Cross-Diameters *&c.* on the Side, take the following Method: — Measure the slant Side of the Vessel, and multiply the Length thereof by the intended Distance of the Cross-Diameters (or Dimensions); divide the Product by the perpendicular Depth of the Vessel, and the Quotient will be the Distance of the Cross-Diameters, *&c.* measured on the slant Side.

Note. It is both more *expeditious* and *certain*, to mark the Sides of any Vessel, where the Dimensions are to be taken, with a Pair of *Compasses* (such as are used by *Coopers*, *&c.*), than by any other Method that has yet occurred to me.

EXAMPLE.

Let the Depth of a Distiller's round Wash-Back be 61.8 Inches, the Drip, or Depth of the Liquor, at the intended Dipping-place 1.2, and the Cross-

* If the Altitude of any straight-sided Vessel be denoted by a and the Difference of the top and bottom Dimensions by d; then it is evident (by similar Triangles) that $d : a :: 1$ (*viz*, one Inch) : $\frac{a}{d}$ = the perpendicular Distance of the Dimensions.

Crofs-Diameters as below; required the Content of this Veffel in Wine Gallons.

Inches.		*Diam.*	*Diam.*	*Area.*	*Gallons.*
12	(6 Inches fr. the Top)	52.8 . .	51.6 . .	9.26 . .	111.12
10	(17 from the Top) . .	53.5 . .	52.5 . .	9.55 .	95.50
10	(27 from the Top) . .	54.3 . .	53.5 . .	9.87 . .	98.70
10	(37 from the Top) . .	55.0 . .	54.4 . .	10.17	. 101.70
10	(47 from the Top) . .	55.9 . .	55.4 . .	10.56 . .	105.60
8.6	(56.3 fr. the Top) . .	56.6 . .	56.3 . .	10.83 . .	93.138
Drip 1.2	. . . .				10.00
Depth 61.8	. . . .			Content	615.758 Wine Gallons.

Grofs Depth	61.8	Gal.
Drip	1.2 . .	10
Neat Depth	60.6	

The Manner of finding the foregoing Areas *&c.* is extremely eafy: Thus, for Inftance, the Sum of the two Crofs-Diameters at the Top being 104.4, the Half of which is 52.2 the Diameter at 6 Inches from the Top of the Veffel; then againft this Diameter, in the Table of Wine Areas, we have 9.26, which being multiplied by 12, gives 111.12 Wine Gallons, the Content for the firft 12 Inches from the Top of the Back. By the very fame Method the other five Areas, *&c.* were obtained.

In Order to form the preceding Work into a Table, whereby the Quantity of Liquor in the Back, at any Number of dry Inches, may be known by Infpection; proceed thus: From the whole Content 615.75, fubtract the Area in the Middle of the firft 12 Inches from the Top (*i. e.* 9.26) and the Remainder 606.49 will fhew the Quantity in the Back at one Inch dry; again, from 606.49 take 9.26, the Remainder 597.23 will be the Quantity in the Back at two Inches dry; and by proceeding in the fame Manner for 10 Inches more, we fhall get the Quantities of Liquor in the Back, at 3, 4, 5, 6, *&c.* to 12 Inches dry.

From

From the Quantity at 12 dry Inches, subtract the 2d Area, *viz.* 9.55, and from the Remainder take again the 2d Area, and so on, till we come to the 2d Inch; then proceed with the 3d, 4th, 5th, and 6th Areas, successively, till we get the Quantity in the Back at 60 dry Inches; from which Quantity take $\frac{6}{10}$ths of the 6th Area, and the Remainder will be 10 Gallons (the Drip) if the Work is right.

Though it may, perhaps, be reckoned more elegant to determine the Measure of the Drip, or Fall, of a Tun, &c. by Computation, than to cover its Bottom (with Water) by a known Measure; yet I cannot but think (because the Inclination of a Tun, *when fixed*, is so *very small)* that the latter Method is far more eligible, both with Respect to Expedition and Exactness, than to make use of a Quadrant, or any other Mathematical Instrument, to determine this *small* Inclination; and afterwards to have the Measure of the Drip to find, by a very troublesome Computation:

The OPERATION *for a* TABLE *of dry Inches, in Gallons,* &c.

Inc.	*Gallons.*	*Inc.*	*Contin.*
Full	615.75	5	569.45
1. *A*	9.26		9.26
1	606.49	6	560.19
	9.26		9.26
2	597.23	7	550.93
	9.26		9.26
3	587.97	8	541.67
	9.26		9.26
4	578.71	9	532.41
	9.26		9.26

Crofs-Diameters as below; required the Content of this Veffel in Wine Gallons.

Inches.		*Diam.*	*Diam.*	*Area.*	*Gallons.*
12	(6 Inches fr. the Top)	52.8	51.6	9.26	111.12
10	(17 from the Top)	53.5	52.5	9.55	95.50
10	(27 from the Top)	54.3	53.5	9.87	98.70
10	(37 from the Top)	55.0	54.4	10.17	101.70
10	(47 from the Top)	55.9	55.4	10.56	105.60
8.6	(56.3 fr. the Top)	56.6	56.3	10.83	93.138
Drip 1.2					10.00
Depth 61.8				Content	615.758 Wine Gallons.

Grofs Depth	61.8	Gal.
Drip	1.2	10
Neat Depth	60.6	

The Manner of finding the foregoing Areas *&c.* is extremely eafy: Thus, for Inftance, the Sum of the two Crofs-Diameters at the Top being 104.4, the Half of which is 52.2 the Diameter at 6 Inches from the Top of the Veffel; then againft this Diameter, in the Table of Wine Areas, we have 9.26, which being multiplied by 12, gives 111.12 Wine Gallons, the Content for the firft 12 Inches from the Top of the Back. By the very fame Method the other five Areas, *&c.* were obtained.

In Order to form the preceding Work into a Table, whereby the Quantity of Liquor in the Back, at any Number of dry Inches, may be known by Infpection; proceed thus: From the whole Content 615.75, fubtract the Area in the Middle of the firft 12 Inches from the Top (*i. e.* 9.26) and the Remainder 606.49 will fhew the Quantity in the Back at one Inch dry; again, from 606.49 take 9.26, the Remainder 597.23 will be the Quantity in the Back at two Inches dry; and by proceeding in the fame Manner for 10 Inches more, we fhall get the Quantities of Liquor in the Back, at 3, 4, 5, 6, *&c.* to 12 Inches dry.

From

From the Quantity at 12 dry Inches, ſubtract the 2d Area, *viz.* 9.55, and from the Remainder take again the 2d Area, and ſo on, till we come to the 2d Inch; then proceed with the 3d, 4th, 5th, and 6th Areas, ſucceſſively, till we get the Quantity in the Back at 60 dry Inches; from which Quantity take $\frac{6}{10}$ths of the 6th Area, and the Remainder will be 10 Gallons (the Drip) if the Work is right.

Though it may, perhaps, be reckoned more elegant to determine the Meaſure of the Drip, or Fall, of a Tun, *&c.* by Computation, than to cover its Bottom (with Water) by a known Meaſure; yet I cannot but think (becauſe the Inclination of a Tun, *when fixed*, is ſo *very ſmall)* that the latter Method is far more eligible, both with Reſpect to Expedition and Exactneſs, than to make uſe of a Quadrant, or any other Mathematical Inſtrument, to determine this *ſmall* Inclination; and afterwards to have the Meaſure of the Drip to find, by a very troubleſome Computation:

The OPERATION *for a* TABLE *of dry Inches, in Gallons, &c.*

Inc.	*Gallons.*	*Inc.*	*Contin.*
Full	615.75	5	569.45
1. *A*	9.26		9.26
1	606.49	6	560.19
	9.26		9.26
2	597.23	7	550.93
	9.26		9.26
3	587.97	8	541.67
	9.26		9.26
4	578.71	9	532.41
	9.26		9.26

Inch.	*Contin.*	*Inch.*	*Contin.*
10	523.15	26	369.65
	9.26		9.87
11	513.89	27	359.78
	9.26		9.87
12	504.63	28	349.91
2. *A.*	9.55		9.87
13	495.08	29	340.04
	9.55		9.87
14	485.53	30	330.17
	9.55		9.87
15	475.98	31	320.30
	9.55		9.87
16	466.43	32	310.43
	9.55	4. *A.*	10.17
17	456.88	33	300.26
	9.55		10.17
18	447.33	34	290.09
	9.55		10.17
19	437.78	35	280.92
	9.55		10.17
20	428.23	36	269.75
	9.55		10.17
21	418.68	37	259.58
	9.55		10.17
22	409.13	38	249.41
3. *A.*	9.87		10.17
23	399.26	39	239.24
	9.87		10.17
24	389.39	40	229.07
	9.87		10.17
25	379.52	41	218.90
	9.87		10.17

Inch.

Inch.	*Contin.*	*Inch.*	*Contin.*
42	208.73	52	103.13
5. *A.*	10.56	6. *A.*	10.83
43	198.17	53	92.30
	10.56		10.83
44	187.61	54	81.47
	10.56		10.83
45	177.05	55	70.64
	10.56		10.83
46	166.49	56	59.81
	10.56		10.83
47	155.93	57	48.98
	10.56		10.83
48	145.37	58	38.15
	10.56		10.83
49	134.81	59	27.32
	10.56		10.83
50	124.25	60	16.49
	10.56		6.49 = $\frac{6}{10}$th of 10.83.
51	113.69	60.6	10.00
	10.56	1.2	10. *Drip.*
		61.8	00.

It is to be obſerved, that, in tabulating any Veſſel, there is no Neceſſity for writing down (as above) the Area at every Inch; but only to enter it on a ſmall Piece of Paper, and move it downwards as we ſubtract, or add, according as the Table is to be made for the dry, or wet Inches: And, that we may proceed with more Certainty, it will be neceſſary to examine the Operation, at every different Area, in the following Manner.

	Gallons.
Whole Content, see *Pa.* 120.	615.75
Subtract 12 times the *top Area*	111.12
Remains, at 12 Inches dry,	504.63
Subtract 10 times the *2d Area*	95.50
At 22 Inches dry	409.13
Subtract 10 times the *3d Area*	98.70
At 32 Inches dry	310.43
Subtract 10 times the *4th Area*	101.70
At 42 Inches dry	208.73
Subtract 10 times the *5th Area*	105.60
At 52 Inches dry	103.13
Subtract 8.6 times the *6th Area*	93.13
Remains the *Drip*, or *Fall*,	10.00

It may be proper to observe here, that, though the whole Content of this Vessel is *strictly* 615.758 Gallons, it was needless, in forming the preceding Table, to carry on the third Decimal (8); since it is very evident, that the Conclusion at 60.6 Inches would have been the very same as is exhibited in the Table: For, at 60 Inches, we should have had 16.498; from which taking 6.498 (*i. e.* $\frac{6}{10}$ths of 10.83), leaves 10 Gallons (the Drip), *as before:* Moreover, instead of 103.13 at 52 dry Inches (as above), we should have had 103.138; from which subtracting 93.138 (*i. e.* 8.6 times the 6th Area), the Remainder will be 10 Gallons; *the same as before.*

It may be also necessary to observe, with Respect to forming a Table for wet Inches, that the Decimal

mal Parts, which may happen in the whole Depth of any Veſſel, muſt be conſidered in the top Area, and the Drip muſt, conſequently, be taken out of the Inches (a whole Number) correſponding to the Bottom Area; by which Means a Table may be formed to two Places of Decimals, and the Operation proved, *ſtrictly true*, at every different Area; which otherwiſe could not be performed, without finding the Content of the Veſſel, and tabulating the ſame, to three Places of Decimals.

Sometimes the Poſition of a Diſtiller's Waſh-Back, *&c.* is ſuch, that it is found neceſſary to fix the Dipping-place thereof, at ſome certain Diſtance above the Top of the Back, which Diſtance is called the *Curb*; and, to avoid unneceſſary Trouble in tabulating the Veſſel, it is always taken a whole Number, in Inches.—Suppoſe, in the preceding Example, there had been a Curb of 9 Inches; then, at the Time of taking the Dimenſions of the Back, we ſhould have written down,

Whole Depth	70.8		
Curb	9.0		
Groſs Depth of the Back	61.8		*Gal.*
Drip	1.2	. .	10
Neat Depth	60.6		

Moreover, in tabulating a Veſſel where there is a Curb, inſtead of beginning at *Full* (as in the preceding Table), we muſt begin as follows:

Inches.

Inches.		*Gallons.*
Curb 9	. .	615.75
10	. .	606.49
11	. .	597.23
12	. .	587.97
		&c. as before.

If the foregoing Dimenſions were thoſe of a Brewer's round Guile-Tun; then the Method of finding its Content, and tabulating the ſame, would differ but little from that above exhibited.

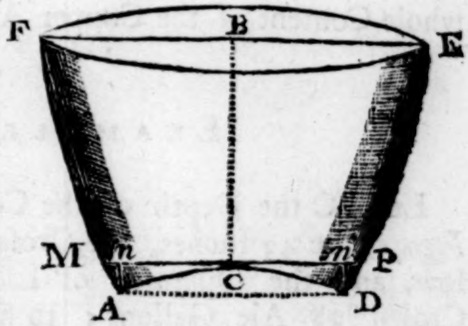
For if from the whole Content, found in Barrels, Firkins, and Gallons (and to two Decimal Places of a Gallon), we ſubtract the top Area, reduced in like Manner; the Remainder will ſhew the Quantity, in Barrels, Firkins, *&c.* in the Tun at one Inch dry: Proceed in the ſame Manner to find the Quantities at the 2d, 3d, 4th, *&c.* dry Inches; and likewiſe with the 2d, 3d, and 4th, *&c.* Areas.—This Method will, I apprehend, be ſufficiently illuſtrated by the following Operation.

To take the Dimenſions of a Copper *with a riſing Crown; to find its Content, and tabulate the ſame in Barrels, Firkins,* &c.

It is well known that Coppers and Stills are always fixed with their Bottoms ſomewhat inclined to the Horizon, their loweſt Part being at the Cock, for more Convenience of draining off the Liquor; but this Inclination being ſo very ſmall, that the Figure of the Surface of the Liquor, at every Altitude, may be conſidered as a Circle without any (ſenſible) Error reſulting therefrom; therefore the Dimenſions may be taken in the following Manner.

Suppoſe

Suppose the Figure ACDEF to represent a Copper when fixed, and A the Place of the Cock; through C, the Center of the Crown, extend a Piece of Pack-



Thread in such a Manner, that the perpendicular Distances A*n* and D*n* may be equal to each other, and let Marks be made, on the Sides of the Copper, at M and P; also extend a small Cord (or Pack-Thread EF) diametrically over the Top of the Copper, and with one End of the Dimension-Cane on the Center C, find the nearest Distance to the said Pack-Thread EF; that Distance (*viz.* BC) will be the internal Altitude of the Copper.

Now let the Copper be quartered, at the Bottom and Top, by the Method already laid down in *Page* 117, for a round Back or Tun, and let Cross-Diameters be taken in the Middle of every 6, 7, 8, 9, 10, *&c.* Inches of the Altitude BC, beginning from the Top; that is, let the first Cross-Diameters be taken, either at 3, 3.5, 4, 4.5, or 5, *&c.* Inches from EF; and the second Cross-Diameters either at 9, 10.5, 12, 13.5, or 15 from the Top, and so on, towards C: Then find, by the Table, the Areas in Ale Gallons, corresponding to those Cross-Diameters, in the same Manner as is laid down for Wine Areas, *Pa.* 119; these Areas being each multiplied by their corresponding Parts of the Depth, and the Sum of the Products, together with the Quantity

which

which *exactly* covers the Crown ACD, will be the whole Content of the Copper ACDEF.

Example.

Let BC the Depth of the Copper (see the last *Figure*) be 43 Inches, the Cross-Diameters as below, and the Quantity of Liquor to cover the Crown 38 Ale Gallons; to find its Content in *Beer Barrels*, *Firkins*, *&c.*

Inches.			Cross-Di.		Half the Sum of the Cross-Diameters
13	(6.5 from the Top)	97.5		98.1	97.8
10	(18 from the Top)	95.8		96.4	96.1
10	(28 from the Top)	94.3		93.9	94.1
10	(38 from the Top)	93.2		93.0	93.1

Therefore, by the Table of Ale (or Beer) Areas, *&c.* the Work will stand as follows.

Inches.	*Diam.*	*Area* in Gal.	*Content* in Gal.	*Area* in B.	F.	G.Pts.	*Content* in B.	F.	G.Pts
13	97.8	26.63	346.19	0	2	8.63	9	2	4.19
10	96.1	25.72	257.20	0	2	7.72	7	0	5.20
10	94.1	24.66	246.60	0	2	6.66	6	3	3.60
10	93.1	24.13	241.30	0	2	6.13	6	2	7.30
	To cover the Crown		38				1	0	2
Dep. 43	The whole Content		1129.29				31	1	4.29

A TABLE *of the preceding Work: The Method of forming of which has been already observed, in Pa.* 126.

Inch.	B.	F.	G.Pts.
Full	31	1	4.29
	1.Ar.2		8.63
1	30	2	4.66
2	29	3	5.03
3	29	0	5.40
4	28	1	5.77
5	27	2	6.14
6	26	3	6.51
7	26	0	6.88
8	25	1	7.25
9	24	2	7.62
10	23	3	7.99
11	23	0	8.36
12	22	1	8.73
13	21	3	0.10
	2.Ar.2		7.72
14	21	0	1.38
15	20	1	2.66
16	19	2	3.94
17	18	3	5.22
18	18	0	6.50
19	17	1	7.78
20	16	3	0.06
21	16	0	1.34
22	15	1	2.62
23	14	2	3.90
	3.Ar.2		6.66
24	13	3	6.24

Inch	B.	F.	G.Pts
25	13	0	8.58
26	12	2	1.92
27	11	3	4.26
28	11	0	6.60
29	10	1	8.94
30	9	3	2.28
31	9	0	4.62
32	8	1	6.96
33	7	3	0.30
	4.Ar.2		6.13
34	7	0	3.17
35	6	1	6.04
36	5	2	8.91
37	5	0	2.78
38	4	1	5.65
39	3	2	8.52
40	3	0	2.39
41	2	1	5.26
42	1	2	8.13
43	1	0	2.00
	Remains to cover the Crown.		

The Method of tabulating a Veſſel, having different Areas at any aſſigned Number of *Inches*, is very eaſy; as in the two preceding Examples: But, when there are any Incumbrances to be deducted out of the groſs Content, ſuch as *Croſs-Bars* (or *Beams*), *Planks*, *&c.* frequently uſed in ſupporting, or repairing, the Sides of a Veſſel, the Operation (to proceed with Certainty) will then be ſomewhat more intricate; which I ſhall endeavour to explain as follows.

Suppoſe, for Example, the mean Length of a Veſſel (in Form of a *rectangular* Parallelopipedon) to be 184.6, the mean Breadth 104.4 Inches, and the Depth and Dimenſions of the Incumbrances as below; required the *neat* Content in Wine Gallons, and the Method of tabulating the ſame.

	Inches.	Gall.
Groſs Depth	80.6	
Drip	2.8 . .	126
Neat Depth	77.8	

Bottom of the 1ſt Croſs-Bar (or Croſs-Beam) from the Bottom of the Back	69.5
Length	99.5
Breadth	3.8
Depth	4.8
Bottom of the 2d + Bar from the Bottom	30.8
Length	99.5
Breadth	3.6
Depth	3.4
Two perpendicular Planks, Length (or Height) of each, beginning at the Bottom . .	71.5
Covered with Water in the Drip	2.0
Neat Length of the Planks	69.5
Breadth	12.5 each.
Thickneſs	2.5 each.

184.6

184.6 = the mean Length.
104.4 = the mean Breadth.

7384
7384
18460

231) 19272.24(83.42 = the groſs Area, *very nearly*.
77.8 = the neat Depth.

66736
58394
58394

6490.076
Add 126. = the Drip.

Gives 6616.076 = the groſs Content.

First Croſs-Bar.

Length 99.5
Breadth 3.8

7960
2985

231)378.10(1.63 = the Area of the 1ſt + Bar.
4.8 = the Depth.

1304
652

7.824 = the Content of the 1ſt + Bar.

Second Crofs-Bar.

Length 99.5
Breadth 3.6

5970
2985

231)358.20(1.55 = the Area of the 2d + Bar.
3.4 = the Depth.

620
465

5.270 = the Content of the 2d + Bar.

Two Planks.

Breadth 12.5
Thicknefs 2.5

625
250

31.25
2

231)62.50(.27 = the Area of both the Planks.
69.5 = the Length.

135
243
162

18.765 = the Content of both the Planks.

First

	Area.	Content.
First + Bar . .	1.63 . .	7.824
Second + Bar . .	1.55 . .	5.270
Planks . .	.27 . .	18.765

Content of the Incumbrances 31.859; which being deducted from 6616.076 the gross Content, leaves 6584.217 for the neat Content of the Vessel.

Now, in Order to tabulate this Vessel with Certainty, proceed in the following Manner.

Neat Depth of the Vessel	77.8
Neat Depth of the Planks	69.5
Top of the Planks from the Top of the Vessel	8.3.

It is very evident, that if the Depth of either Cross-Bar be added to the Distance of its Bottom from the Bottom of the Vessel, and that Sum subtracted from the gross Depth, the Remainder will be the Distance of the Top of that Cross-Bar from the Top of the Vessel.

Hence it follows, that the Distance of the Top of the first Cross-Bar from the Top of the Vessel is 6.3; and that of the Top of the second Cross-Bar from the Top is 46.4 Inches.

Therefore the following Method, or one similar to it, must be laid down, as preparatory to the forming of this (or such like) Vessel into a Table of dry Inches, in Order to take out the Incumbrances (let them happen how they will) in their proper Places, *strictly true.*

Names

Names of the Incumbrances taken out.	Areas of the Incumbrances,	Grofs Area.	Neat Area.	Depth.	Gallons.
Mean Length . . . . Mean Breadth . . .	184.6 104.4	83.42	. . .	6.3	525.546
Firft Crofs Bar . . .	1.63	83.42 1.63	81.79	2.0	163.580
Firft Crofs-Bar and Planks	1.63 .27	83.42 1.90	81.52	2.8	228.256
Planks	.27	83.42 .27	83.15	35.3	2935.195
Second Crofs-Bar and Planks .	1.55 .27	83.42 1.82	81.60	3.4	277.440
Planks	.27	83.42 .27	83.15	28.0	2328.200
		Drip . .	. . .	2.8	126.
			Depth	80.6	6584.217 Content.

It will be unneceffary to form the preceding Work into a Table of dry Inches, fince the Method of Operation is, *very nearly*, the fame as that in *Pa.* 121; the only Difference, or Difficulty, will happen at the Alteration of the different Areas; which I fhall here endeavour to explain.

From the whole Content 6584.217 fubtract the grofs Area 83.420, and from the Remainder take again 83.420, and fo on, 6 times fucceffively; then, for the Quantity at 7 Inches, proceed thus: — To $\frac{3}{10}$ths of the Area 83.42 add $\frac{7}{10}$ths of 81.79, this Sum being fubtracted from the *Quantity* at 6 dry Inches, leaves *that* at 7 dry Inches: Alfo for the 9th Inch, we muft fubtract, from the Quantity at 8 Inches, the Sum of $\frac{3}{10}$ths of 81.79 and $\frac{7}{10}$ths of 81.52; and for the 12th Inch, fubtract the Sum of $\frac{1}{10}$th of 81.52 and $\frac{9}{10}$ths of 83.15, *&c.* This will appear very evident from the following Method; which Method (fee *Pa.* 124) will be found very ufeful in proving your Work.

Whole

Whole Content	6584.217
Subtract	525.546
Leaves, at 6.3 In. dry,	6058.671
Subtract	163.580
At 8.3 Inches	5895.091
Subtract	228.256
At 11.1 Inches	5666.835
Subtract	2935.195
At 46.4 Inches	2731.640
Subtract	277.440
At 49.8 Inches	2454.200
Subtract	2328.200
At 77.8 Inches	126 = the *Drip*, or *Fall*.

If the Diſtances of the Tops of the *Croſs-Bars* from the Top (or Bottom) of the Veſſel, and their *Depths*, are *all* taken (or *ſuppoſed*) whole Numbers; the Operation, for forming a Table of dry (or wet) Inches, will then be greatly ſhortened: But it is certainly much more maſterly, and ſatisfactory to an inquiſitive Mind, to tabulate the Content of ſuch a Veſſel, from the Diſtances and Dimenſions of Croſs-Bars as they are *actually* taken, than to *ſuppoſe* them .3, .4 or perhaps $\frac{5}{10}$ths more or leſs than the Truth, in Order to avoid a little more Trouble in the Computation.

Prob.

PROP. VI.

To find the Content in Ale and Wine Gallons, and Malt Bushels, of a Vessel (called a Prismoid) *whose parallel Ends are any dissimilar Rectangles, and the Sides of it are four plane Surfaces.*

GENERAL RULE.

To the longest (or shortest) Side of the Rectangle at either End, add that Side at the other End (whether it be the Length or Breadth) which is parallel to it; multiply this Sum by the Sum of the other two parallel Dimensions (*i. e.* one at the Top and the other at the Bottom), and to the Product add the Areas of the two Ends; this Total being multiplied by the Height (or perpendicular Altitude), and the Product thereof divided by 1692 for Ale, 1386 for Wine Gallons, and 12902.5 for Malt Bushels, gives the Content sought.*

EXAMPLE.

Suppose there is a Tun, whose parallel Ends are Rectangles, the Length and Breadth of the Top 36 and 32, the Length and Breadth of the Bottom (being, in this Case, respectively parallel to *those* above) 48 and 40, and the Height 60 Inches; required the Content of the Tun in Ale Gallons, *&c.*

OPERATION.

---

* There is a very elegant Investigation given in *Simpson*'s Fluxions, 2*d Ed.* Pa. 170, for determining the Content of a Prismoid, when the Sides of the Rectangle at one End, are less than the parallel Sides of the *other:* And from the same Method of Reasoning (supposing *a* and *b* to denote the Length and Breadth of the Rectangle at one End, *c* and *d* the Sides of the Rectangle at the *other*, parallel to those of *a* and *b* respectively), it will appear that the Theorem which that illustrious Author has there given is general, let the rectangular Ends be what they will.

OPERATION.

The Length at the greater End	48
The Length at the other End which (in this Case) is parallel to *that* above	36
Sum	84
The Sum of the other two parallel Dimensions, or Breadths (in this Case)	72
	168
	588
Product	6048
The Area of the greater End (*i. e.* 48 multiplied by 40) . . .	1920
The Area of the less End (*i. e.* 36 multiplied by 32)	1152
Total	9120
Multiplied by the Height	60
	547200

1692)547200.00(323.41 Ale Gallons.
1386)547200.00(394.80 Wine Gallons.
12902.5)547200.00(42.4 Malt Bushels.

Some Authors have asserted, that the same Rule which gives the Content of a Prismoid, will also hold good in any straight-sided Vessel, whose parallel Ends are dissimilar Ellipses, and any-how posited; or if one End is an Ellipsis and the other a Circle:* But it appears that this Assertion

T is

* Let the Ellipsis ABCD (*Fig.* VI.) represent the Base of the Solid, and the Circle *dcgf* the Top thereof; also let *beap* represent a Plane of the Section of the Solid, cut any-where parallel to its Ends: Then, drawing a Diameter

is without proper Foundation, and seems to have arisen wholly from the following Supposition; namely,

---

Diameter EF, it is very evident, (because the Sides of the Solid in every Plane conceived to pass through the Centers of the Ellipsis and Circle are Right-lines) that it will be, as $Be : ed :: Em : mb$ ($:: aC : ac :: Dp : pg$, &c.) let the Position of the Diameter be how it will; or, by Composition, $Be+ed : ed :: Em+mb : mb$; $\therefore Bd : de :: Eb : mb$. Now let the given Semi-transverse $OC=a$, the Semi-conjugate $OB=b$, the Radius $Ob=r$, $Od=d$, the Abscissa $na=x$, and the Ordinate $mn=y$: Moreover, let Bd be to de (or Eb to mb), in every Position of the Diameter EF, as 1 to n: Then $d-x=On$, and $\sqrt{\overline{d-x}|^2+y^2}=Om$; $\therefore \sqrt{\overline{d-x}|^2+y^2}-r=mb$, whence $n : 1 :: \sqrt{\overline{d-x}|^2+y^2}-r$: $\dfrac{\sqrt{\overline{d-x}|^2+y^2}-r}{n}=Eb$, consequently $\dfrac{\sqrt{\overline{d-x}|^2+y^2}-r}{n}+r=OE$; then, by similar Triangles, $\sqrt{\overline{d-x}|^2+y^2}\ (Om)$: $\dfrac{\sqrt{\overline{d-x}|^2+y^2}-r+rn}{n}\ (OE) :: d-x\ (On) : \dfrac{\overline{d-x}\times\sqrt{\overline{d-x}|^2+y^2}-r+rn}{n\sqrt{\overline{d-x}|^2+y^2}}$ $\left(=\dfrac{d-x}{n}-\dfrac{\overline{r-rn}\times\overline{d-x}}{n\sqrt{\overline{d-x}|^2+y^2}}\right)=OG$; again, by similar Triangles, $\sqrt{\overline{d-x}|^2+y^2}\ (Om)$: $\dfrac{\sqrt{\overline{d-x}|^2+y^2}-r+rn}{n}\ (OE) :: y\ (mn)$: $\dfrac{y\sqrt{\overline{d-x}|^2+y^2}-\overline{1-n}\times ry}{n\sqrt{\overline{d-x}|^2+y^2}}$ $\left(=\dfrac{y}{n}-\dfrac{\overline{1-n}\times ry}{n\sqrt{\overline{d-x}|^2+y^2}}\right)=EG$; but, by the Property of the Ellipsis, EG^2 $\left(=\dfrac{OB^2}{OA^2}\times\overline{AO+OG}\times\overline{AO-OG}\right)=\dfrac{b^2}{a^2}\times a^2-\overline{\dfrac{d-x}{n}-\dfrac{\overline{r-rn}\times\overline{d-x}}{n\sqrt{\overline{d-x}|^2+y^2}}}\Bigg|^2$; $\therefore$ $\overline{\dfrac{y}{n}-\dfrac{\overline{1-n}\times ry}{n\sqrt{\overline{d-x}|^2+y^2}}}\Bigg|^2=\dfrac{b^2}{a^2}\times a^2-\overline{\dfrac{d-x}{n}-\dfrac{\overline{r-rn}\times\overline{d-x}}{n\sqrt{\overline{d-x}|^2+y^2}}}\Bigg|^2$, the Equation of the Curve *beap*; which, as it returns into itself by the Nature of the Section, is of the Oval Kind.

COROLLARY.

Hence it appears, that, if $r=0$, the above Equation becomes $\dfrac{b^2}{a^2}\times$

a^2.

namely, *if in any Prismoid another straight-sided Vessel (or Solid) be inscribed, whose Ends are Ellipses, the Transverse and Conjugate Diameters of each, respectively equal to the Lengths and Breadths of the said Prismoid; that then the Sections parallel to the Ends*

$\overline{a^2 - \frac{\overline{d-x}|^2}{n^2}} = \frac{y^2}{n^2}$; or, by substituting $\frac{d^2}{a^2}$ for n^2 (because, in that Case, $na=d$, for $n : 1 :: d-r : a-r$), we shall then have $\frac{b^2}{a^2} \times \overline{a^2 - \overline{d-x}|^2} \times \frac{a^2}{d^2} = \frac{a^2}{d^2} \times y^2$; whence $y^2 = \frac{b^2}{a^2} \times \overline{2dx - x^2}$, answering to the Property of an Ellipsis, and similar to the given one ABCD: But if $n=1$, or mn coincides with EG, then $d=a$, and therefore the last Equation becomes $y^2 = \frac{b^2}{a^2} \times \overline{2ax - x^2}$, answering to the given Ellipsis ABCD.

SCHOLIUM.

The Nature of the Curve *beap* is the very same as that which may be conceived to be described about an Ellipsis, similar to the given one ABCD, in such a Manner, that the Distance between the two Curves, measured in the Radius-Vector, may be every-where equal to a constant Quantity: For let OE (*Fig.* VI.) be $=$R, and bE$=$R$-r$: Then $1 : n :: R-r : \overline{R-r} \times n = bm$; $\because nR - rn + r (= nR + \overline{1-n} \times r) = Om$ the Radius-Vector; whence it is plain that the first Term (nR) expresses the Radius of an Ellipsis similar to ABCD, and the second Term ($\overline{1-n} \times r$) is a constant Quantity: But it may be proved in a general Manner, supposing two concentric Ellipses described, having the Difference of the Conjugates equal to the Difference of their Transverse Diameters, that the Distance between the two Elliptic Arcs, measured in any other Diameter, is greater than the Difference of the Semi-conjugates, or Semi-transverses.

By the same Method of Reasoning, as in the preceding general Investigation, it will be found that the Figure of the Section, parallel to the Ends of the Solid, can *never* be an Ellipsis, unless the said parallel Ends were similar Ellipses, and similarly posited; *i. e.* the Transverse and Conjugate Diameter of each End, respectively parallel to one another; which Circumstance can only obtain when the Solid is the Frustum of an Elliptic Cone: I shall only farther add, that the Curve (forming the Figure of the Section) will be of the same Order, whether the Ends of the Solid are dissimilar Ellipses, and similarly posited; or similar Ellipses, and dissimilarly posited; *i.e.* the Transverse Diameter of one End parallel to the Conjugate of the other: But if the parallel Ends of the Solid are dissimilar Ellipses, and so posited that neither the Transverse nor Conjugate Diameter of one End is parallel to *those* of the other End; then the Equation of the Curve, of the forementioned Section, will be the most complex.

of such a Solid, will also be Ellipses:—But, that this cannot be the Case, the preceding *Note* will (I make no Doubt) sufficiently convince every judicious Reader.—It may, however, be proper to observe, that the Content of a Tun of this Form may be obtained with the greatest Expedition, by the general Rule laid down farther on (derived from the Property of equidistant Perpendiculars, *&c.*); and the Result will be sufficiently exact, if the Section in the Middle be considered as an Ellipsis: For the said Rule is *strictly true* in every *straight-sided* Vessel; provided the Measure of the two Ends, and that of a parallel Section in the Middle, can be *truly* determined. — This will be demonstrated farther on.

PROP. VII.

To find the Content of a Sphere in Ale and Wine Gallons, and Malt Bushels.

RULE.

The Cube of the Diameter of the Sphere being

$$\text{multiplied by}\left\{\begin{array}{l}.0018567\\.0022666\\.0002433\end{array}\right\}\left\{\begin{array}{l}\text{for Ale Gal.}\\\text{Wine Gal.}\\\text{Malt Bush.}\end{array}\right\}\text{, or divided}$$

$$\text{by}\left\{\begin{array}{r}538.58\\441.17\\4107.00\end{array}\right\}\left\{\begin{array}{l}\text{for Ale Gall.}\\\text{Wine Gall.}\\\text{Malt Bush.}\end{array}\right\}\text{, will give the Con-}$$

tent sought.

EXAMPLE.

Required the Content of a Sphere (in Ale Gallons, *&c.*) whose Diameter is 48 Inches.

OPERATION.

$$\begin{array}{r} 48 \\ 48 \\ \hline 384 \\ 192 \\ \hline 2304 \\ 48 \\ \hline 18432 \\ 9216 \\ \hline \text{The Cube of the Diam. } 110592 \\ \hline \end{array}$$

538.58)110592.0000(205.34 Ale Gallons.
441.17)110592.0000(250.22 Wine Gallons.
4107)110592.00 (26.92 Malt Bushels.

By the Sliding-Rule.

To $\left\{\begin{array}{l}23.2\\21.0\\64.1\end{array}\right\}$ on D, set 48 on C; then against 48 on D, we shall have $\left\{\begin{array}{l}205.34\\250.22\\26.92\end{array}\right\}$ the above Contents on C.

Because the Content of every Sphere is $\frac{2}{3}$ds of its circumscribing Cylinder, it is evident that the above Multipliers must be only $\frac{2}{3}$ds of those for the Cylinder (see *Pa.* 83), and consequently the Divisors for the Cylinder, must be likewise $\frac{2}{3}$ds of those for the Sphere.

There

There is another Way (besides that given above) of solving this Question by the *Sliding-Rule*, deduced from *Prop.* 6, *Pa.* 50, to which I refer; and only observe here, that the three given Numbers (in this Case) are

$$\left\{\begin{array}{l} 1,\ 48 \text{ and } .0018567 \\ 1,\ 48 \text{ and } .0022666 \\ 1,\ 48 \text{ and } .0002433 \end{array}\right\} \begin{array}{l} \text{for Ale Gallons.} \\ \text{Wine Gallons.} \\ \text{Malt Bushels.} \end{array}$$

PROP. VIII.

The Transverse and Conjugate Axes of a Spheroid being given; to find its Content in Ale Gallons, &c. (See Def. 29, *Pa.* 59).

RULE.

Multiply the Square of the Conjugate, by the Transverse Axis; and that Product being multiplied, or divided, as in the last Proposition, gives the Content sought.

EXAMPLE.

Let the Conjugate Axis of a Spheroid be 24.5 and the Transverse 38 Inches; required its Content in Ale and Wine Gallons, and Malt Bushels.

OPERATION.

OPERATION:

$$
\begin{array}{r}
24.5 \\
24.5 \\
\hline
1225 \\
980 \\
490 \\
\hline
\end{array}
$$

The Sq. of the Conjugate 600.25
Transverse 38

$$
\begin{array}{r}
480200 \\
180075 \\
\hline
\end{array}
$$

538.58)22809.5000(42.35 Ale Gal.
441.17)22809.5000(51.7 W. Gal.
4107)22809.50 (5.55 M. Bush.

Because every Spheroid is equal to $\frac{2}{3}$ds of its circumscribing Cylinder; therefore the Method of Operation is the same as that for the Sphere.

By the Sliding-Rule.

To $\left\{\begin{array}{l}23.2\\21.0\\64.1\end{array}\right\}$ on D, set 38 on C; then opposite 24.5 on D, we shall have $\left\{\begin{array}{l}42.35\\51.7\\5.55\end{array}\right\}$ the above Content on C.

PROP. IX.

The Altitude of the Segment of a Sphere, and the Diameter of its Base being given, to find its Content in Ale Gallons, &c.

RULE.

RULE.

To the Square of the Altitude of the Segment, add three times the Square of half the Diameter of its Base; multiply this Sum by the Altitude, and the Product being multiplied, or divided, as in the preceding Propositions, gives the Content required.*

EXAMPLE.

Suppose AB the Altitude of the Segment ACD to be 12, and CD the Diameter of the Base 32 Inches; required its Content in Ale and Wine Gallons, and Malt Bushels.

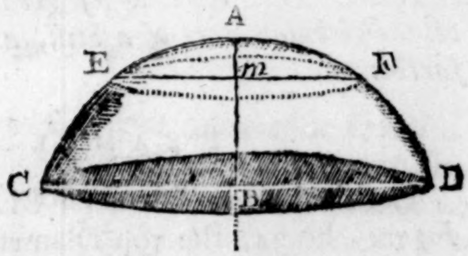
OPERATION.

---

* Let the Diameter CD (*Fig.* VII.) $= a$ ($p = 3.1416$), CE $= x$, and DE $= d - x$: Then EF^2 ($=$ ED $\times$ EC) $= ax - x^2$; whence the Fluxion of the Segment FCH is $pax\dot{x} - px^2\dot{x}$, the Fluent whereof is $\frac{pax^2}{2} - \frac{px^3}{3}$, or $px^2 \times \frac{3a - 2x}{6}$, the Content of the Segment FCH: But, supposing EF to be denoted by d, we have, by similar Triangles, $a \times x = x^2 + d^2$ (FC^2); $\therefore a = \frac{x^2 + d^2}{x}$; which being substituted (above) for a, we get $p \times \frac{3x^3 + 3dx^2}{6} - \frac{2x^3}{6}$, or $\frac{px}{6} \times \overline{x^2 + 3d^2} =$ the Solidity of the Segment FCH; but $\frac{p}{6} = .5236$, $\because \frac{282}{.5236} = 538.58$, and $\frac{.5236}{282} = .0018567$. Q. E. I.

COROLLARY.

Hence, if x be taken $= a$ ($=$ the whole Diameter CD), then is $d = 0$; and consequently the above Expression becomes $\frac{pa^3}{6} =$ the Content of the whole Sphere: But the Content of a Cylinder, whose Diameter and Height each $=$ CD, is expressed by $\frac{pa^3}{4}$; whence it is evident that a Sphere is two-thirds of its circumscribing Cylinder.



OPERATION.

The Square of the Altitude	144
3 times the Square of (16) the Semi-base	768
Sum	912
Altitude	12
	1824
	912
	10944

538.58)10944.0000(20.32 Ale Gallons.
441.17)10944.0000(24.80 Wine Gallons.
4107)10944.0000(2.66 Malt Bushels.

PROP. X.

The top and bottom Diameters, and the Altitude of the Frustum of a Sphere being given; to find its Content in Ale Gallons, &c.

RULE.

To half the Sum of the Squares of the top and bottom Diameters, add $\frac{2}{3}$ds of the Square of the Altitude; this Sum being multiplied by the Altitude, and the Product divided by 359, 294, and 2738, will give the Content in Ale and Wine Gallons, and Malt Bushels respectively.*

U Note.

* This Rule is very elegantly investigated in *Simpson*'s Fluxions, *1st Ed. Pa.* 211.

Note. This Rule is of great Use, in gauging of the globular Part of a Still, as will be exemplified farther on.

EXAMPLE.

Let the bottom Diameter CD (see the *preceding Figure*) be 34, the top Diameter EF 16.5, and the Altitude B*m* 7.2 Inches; to find the Content of the Frustum CEFD in Ale Gallons, *&c.*

OPERATION.

Bottom Diam.		Top Diam.		Alt.
	34		16.5	7.2
	34		16.5	7.2
	136		825	144
	102		990	504
			165	
	1156			51.84
			272 25	2
		Add	1156	
				103.68
The Sum of the Squares of the top and bottom Diam.			1428.25	34.56

Half of which is 714.125
$\frac{2}{3}$ds of the Square of 7.2 is 34 56

Sum 748.685
Altitude 7.2

1497370
5240795

Product 5390.5320

359)5390.53(15.01 Ale Gallons.
294)5390.53(18.33 Wine Gallons.
2738)5390.53(1.90 Malt Bushels.

PROP.

Prop. XI.

The Transverse and Conjugate Axes of a Spheroid being given, and also the Height of a Segment thereof; to find its Content in Ale Gallons, &c. (*See* Def. 32, Pa. 60).

Rule.

Divide the Product, contained under the Conjugate Axis and the Altitude of the Segment, by the Transverse Axis, multiply the Square of that Quotient by the Difference between three times the Transverse Axis and twice the Altitude of the Segment; this Product being multiplied, or divided, as in the Sphere (*Pa.* 140), gives the Content sought.*

Example.

Suppose the Transverse Axis AB of a Spheroid to be 80, the Conjugate CD 60, and the Altitude

U 2 B*n*,

---

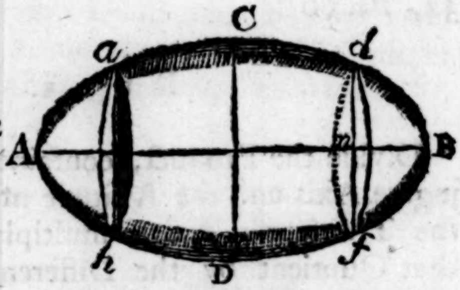
* It is proved, by the Writers on Fluxions, if the Diameter (or Axis) about which the Spheroid is supposed to be generated, be put $= a$, the other Diameter $= b$, and $p = 3.1416$, that the Measure of a Segment, whose Altitude is $= x$, will be expressed by $\frac{pb^2}{a^2} \times \overline{\frac{ax^2}{2} - \frac{x^3}{3}}$, or $\frac{p}{6} \times \frac{b^2x^2}{a^2} \times \overline{3a-2x}$: But $\frac{p}{6} = .5236$, and $\frac{.5236}{282} = .0018567$; therefore $.0018567 \times \left.\overline{\frac{bx}{a}}\right|^2 \times \overline{3a-2x}$ (or $\frac{\left.\overline{\frac{bx}{a}}\right|^2 \times \overline{3a-2x}}{538.58}$) will express the Measure of a Segment in Ale Gallons. Q. E. I.

Corollary.

If x be taken $= a$, we shall have $.0018567 \times ab^2$ for the Measure of the whole Spheroid, which is two-thirds of $.0027851 \times ab^2$, the Measure of a Cylinder (in Ale Gallons) whose Diameter is b, and Altitude a.

B*n*, of the Segment *dBf*, to be 16 Inches; required its Content in Ale and Wine Gallons, and Malt Bushels,

OPERATION.



Alt. of the Seg. 16
Conjugate 60

80)960(12
80

160
160

Three times AB is 240
Twice B*n* is 32

Difference 208
Multiply by the Sq. of 12 (found above) 144

832
832
208

Product is 29952

538.58)29952.0000(55.61 Ale Gallons.
441.17)29952.0000(67.91 Wine Gallons.
4107)29952.00 (7.29 Malt Bushels.

PROP. XII.

The Altitude, and the Diameter of the Base of a Parabolic Conoid being given; to find its Content in Ale Gallons, &c. (*See* Def. 30, Pa. 59).

RULE.

RULE.

The Square of the Diameter of the Baſe, being multiplied by half the Altitude; and the Product divided by 359, 294, and 2738, gives the Content in Ale and Wine Gallons, and Malt Buſhels reſpectively.

EXAMPLE.

Suppoſe the Diameter of the Baſe to be 68.5, and the Altitude 42 Inches; required the Content in Ale Gallons, *&c.*

OPERATION.

```
                          68.5
                          68.5
                     ---------
                          3425
                         5480
                        4110
                     ---------
The Square of the Diam. 4692.25
      ½ the Altitude . . .  21
                     ---------
                        469225
                       938450
                     ---------
 359)98537.25(274.47 Ale Gal.
 294)98537.25(335.16 W. Gal.
2738)98537.25(35.98M.Buſhels.
```

PROP. XIII.

The top and bottom Diameters, and the Altitude of the Fruſtum of a Parabolic Conoid being given; to find its Content in Ale Gallons, &c.

RULE.

RULE.

The Sum of the Squares of the top and bottom Diameters being multiplied by half the given Altitude; and the Product divided by 359, 294, and 2738, gives the Content in Ale and Wine Gallons, and Malt Bushels respectively.

EXAMPLE.

Let the greater Diameter of the Frustum of a Parabolic Conoid be 45, the less Diameter 27, and the Altitude 40 Inches; what is the Content thereof in Ale Gallons, &c.

OPERATION.

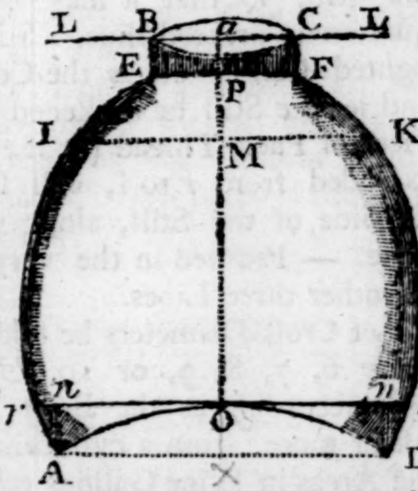
$$\begin{array}{rr} 45 & 27 \\ 45 & 27 \\ \hline 225 & 189 \\ 180 & 54 \\ \hline 2025 & 729 \end{array}$$

$$\begin{array}{rr} & 2025 \\ \text{Add} & 729 \\ \hline \left.\begin{array}{r}\text{The Sum of the Squares} \\ \text{of the Diameters}\end{array}\right\} & 2754 \\ \tfrac{1}{2}\text{ the Altitude} \ldots & 20 \\ \hline & 55080 \end{array}$$

359)55080.00(153.42 Ale Gallons.
294)55080.00(187.34 Wine Gallons.
2738)55080.00(20.11 Malt Bushels.

To

To take the Dimensions of a STILL, *and to find its Content in Wine Gallons; and also a Method for tabulating the same, to every Inch of its whole Depth.*



Suppose ABCDO to represent a Still when fixed, and A the Place of the Cock; upon O, the Center of the Crown, (which is easily distinguished, by a small Dint made by the Copper-Smith), apply the *Sliding-Piece** so that the perpendicular Distances A*n* and D*n* may be equal to each other, and let Marks be made at *r* and *s*, as already described in the Gauging of a Copper, *Pa.* 126; then lay a straight Rule (or Rod LL) diametrically over the Top of the Collar of the Still, and with the End of the Dimension-Cane on the Center O, find the nearest Distance to the said Rule LL, and that Distance (*viz. a*O) will be the whole Depth; from which, the Depth of the Collar being subtracted, we shall obtain PO the internal Depth of the Still: Now, the Dimension-Cane being kept exactly in the Position *a*O, with the Help of your Assistant, let the Plumb-Line be extended as a Diameter IK, exactly at the Seam which is formed by joining the globular Part to the Body of the Still; and measure the Distance MO, which being taken from the whole internal Depth PO, leaves PM the Altitude of the globular Part IEFK.

Quarter

Quarter the Still at the Bottom, and alſo at the Altitude OM, by the Method laid down (*Pa.* 118) for a circular Veſſel: Then in Order to draw four chalk Lines up the Sides of the Still, ſo that it may be every-where truly quartered, proceed thus. Stick a ſmall Piece of lighted Candle on *m* the Center of the Crown, and let the Still be darkened at the Top; then a Piece of Pack-Thread (or the Plumb-Line) being extended from *r* to I, will form a Shadow on the Side of the Still, along which draw a chalk Line. — Proceed in the very ſame Manner for the other three Lines.

Let Croſs-Diameters be taken in the Middle of every 6, 7, 8, 9, or 10, *&c.* Inches, more or leſs, according as the Body of the Still differs, leſs or more, from a cylindrical Form; then find the Areas in Wine Gallons correſponding to theſe Diameters, multiply the Sum of the Areas by their common Diſtance aſunder (ſee *Pa.* 117), and the Product will give the Content of the Body of the Still *r*IK*s*; to which add the Content of the globular Part IEFK (found by *Prop.* 10. *Pa.* 145), and alſo the Quantity which *exactly* covers the Crown, and we ſhall then obtain the whole Content of the Still AEFDO.

Note. It will be always proper to take an even Number of Areas in every Veſſel, whoſe greateſt Diameter is at the Middle of its Length; ſuch as *Caſks*, *Stills*, *&c.* otherwiſe one of the Dimenſions will fall in the Middle, by which Means ſuch Veſſels would be over-gauged.

Note.

Note. I cannot but acknowledge my Obligations to the late Mr. *Thomas Stephens*,* Inſpector-General of the LONDON *Diſtillery*, for the above Method of drawing the four chalk Lines up the Sides of the *Still*; and alſo for communicating to me, ſome uſeful and ingenious Hints, relative to taking the *Dimenſions* of this, and other Veſſels.

Let the Altitude PM of the globular Part (ſee the preceding Figure) be 9 Inches; and let the Croſs-Diameters at the Top and Bottom of the ſaid globular Part, and alſo *thoſe* taken in the Body of the Still be as follow.

Inches.	*Croſs-Diameters.*			*Gallons.*
Alt. PM 9.0	27.0 . . 27.0 56.3 . . 56.2	Content of the globular Part, found by *Prop.* 10. *Pa.* 145.		61.21
			W. Areas.	
9	59.1 . . 60.3	. . .	12.11	108.99
9	63.8 . . 64.1	. . .	13.88	124.92
9	64.2 . . 64.5	. . .	14.05	126.45
10.6	62.1 . . 62.5	. . .	13.99	148.29
			To cover the Crown	36.00
Depth GM 46.6			The whole Content	605.86
				W. Gallons.

Groſs Depth	49.8
Collar	3.2
Internal Depth	46.6
Depth of the Body	37.6
Altitude of the globular Part	9.0
To cover the Crown	36 Gallons.

X The

* The *Sliding-Piece* (*Pa.* 151, contrived by this Gentleman) is very uſeful in taking the Croſs-Diameters of a *Copper*, or *Still*. It conſiſts of two Rods ſliding one by the other, in the ſame Manner as a Pair of Calipers, and when drawn out its full Length (ſee *Fig.* VIII. in the Plate) is 62 Inches: On one Side are graduated Inches and Tenths, and on another are the correſponding Wine Areas; at about one and two Inches from each End (more or leſs, according to the Size of ſuch Inſtrument) are two equal ſquare Holes, to which are fitted two ſmall Pieces to ſlide therein, marked with Inches and Tenths from the Bottom (ſee the *Figure*); theſe ſerve to take a true Diameter directly upon the Center of the Crown, as *rs*. (See the *Fig. Pa.* 151).

The Method of tabulating the Body of a Still (or indeed any Veſſel which is ſuppoſed to have one common Area, or a certain Number of different ones) is the very ſame as that given for a Diſtiller's *Waſh-Back*, *Pa.* 120; to which it may be proper to refer. — But, to determine the *true* Quantity upon every Inch of the globular Part, we muſt previouſly find the Square of the Semi-diameter of that Sphere to which the ſaid globular Part correſponds; in Order thereto, obſerve the following general

RULE.

Divide the Difference of the Squares of half the top and bottom Diameters by twice the Altitude of the Fruſtum, from the Quotient ſubtract half the ſaid Altitude, and the Remainder will be the Diſtance between the Middle of the greater Diameter and the Center of the Sphere;* the Square of

* Let PF ($=\frac{1}{2}$ the leſs Diameter, ſee the following Fig.) $=a$, KM ($=\frac{1}{2}$ the greater Diameter) $=b$, the Altitude PM $=c$, and the required Diſtance OM $=z$; whence (by 47. *Eu.* 1.) $c^2+2cz+z^2+a^2=z^2+b^2$,

$$\therefore z=\frac{b^2-a^2-c^2}{2c}=\frac{b^2-a^2}{2c}-\frac{c}{2}.$$ Q. E. I.

LEMMA.

If the Terms of any Arithmetical Progreſſion (either aſcending or deſcending) be ſquared, and diſpoſed of in a Series; then will the Differences of every two adjacent Terms of that Series, form another Arithmetical Progreſſion, whereof the common Difference will be expreſſed by twice the Square of the common Difference of the firſt Progreſſion.

For any Arithmetical Progreſſion, whoſe firſt Term is m, and the common Difference n, will be expreſſed by m, $+m\pm n$, $+m\pm 2n$, $+m\pm 3n$, $+m\pm 4n$, $+$ &c. whereof the Square of each Term is, m^2, $+m^2\pm 2mn+n^2$, $+m^2\pm 4mn+4n^2$, $+m^2\pm 6mn+9n^2$, $+m^2\pm 8mn+16n^2$, $+$ &c. therefore

The Differences of the two adjacent Terms will form the following Series, *viz.* $\pm 2mn+n^2$, $\pm 2mn+3n^2$, $\pm 2mn+5n^2$, $\pm 2mn+7n^2$, $+$ &c. the common Difference of which, is evidently $2n^2$. Q. E. I.

COROLLARY.

of which being added to the Square of half the greater Diameter, gives the Square of the Semi-diameter of the Globe sought. (*See the following Figure*).

In the preceding Example EP (or PF) is 13.5

```
                                          13.5
                                          13.5
                                         -----
                                           675
                                          405
                                         135
                                        ------
The Square of ½ the top Diameter        182.25

IM (or MK) is 28.125, and twice PM is 18.
              28.125
             -------
              140625
             56250
            28125
           225000
           56250
          -----------
The Sq. of ½       }
the bott. Diam.    } 791.015625
        Subtract     182.25
          -----------
      18)608.765625(33.82
                     4.5 = ½ PM.
                    -----
                    29.32 = MO.
      Add PM         9.00
                    -----
      Gives PO      38.32
           X 2
```

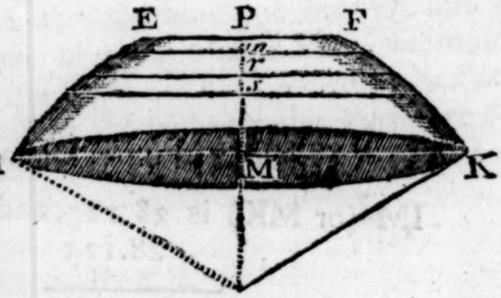
Then

COROLLARY.

If, instead of the Differences of the adjacent Terms of the 1st Progression, we dispose of the Differences of the 1st and 3d, the 2d and 4th, the 3d and 5th, the 4th and 6th, &c. Terms into a Series; we shall then have $\pm 4mn + 4n^2$, $\pm 4mn + 8n^2$, $\pm 4mn + 12n^2$, $\pm 4mn + 16n^2$, &c. the common Difference whereof (instead of $2n^2$) is manifestly $4n^2$; that is, four times the Square of the common Difference of the first Progression.

Then the Square of 29.32 (MO) is 859.6624
Add the Square of 28.125 (KM) 791.0156

Gives the Sq. of the Semi-diam. OK (OI) 1650.6780



Having obtained the Sq. of the Semi-diameter of the Sphere, whereof the Segment IEFK is a Part; then, in Order to inch it down, observe the following Method.

1. To half the Square of the less Diameter EF, add twice the Difference between the Square of the Semi-diameter OK, and the Square of O*m* (*viz.* PO lessened by one Inch), to this Sum add .6666 *&c.* (*i. e.* $\frac{2}{3}$); multiply this last Sum by .0034, and the Product will be the true Content of the first Inch, in Wine Gallons.

2. From the Square of the Semi-diameter OK subtract the Square of O*r* (*viz.* PO less two Inches), and from twice the Remainder take half the Square of the top Diameter EF, multiply the Remainder by .0034; then add this *Product* to the Quantity upon the 1st Inch (found as above), and the Sum will give the true Measure of the 2d Inch.

3. Let the last mentioned *Product* be *reserved*, from which take .0272,† *reserving* the *Difference*, which

† Let I*ab*K (*Fig.* IX.) represent the Frustum of a Sphere, O its Center; draw OM perpendicular to *ab*; take M*m*=1, M*r*=2, MS=3, and M

which being added to the Quantity contained in the 2d Inch (found as above), gives the Quantity for the 3d Inch: Again, from the *reserved Difference* take .0272, add the Remainder to the 3d Inch, the Sum will give the Quantity for the 4th Inch: Proceed in the same Manner to find the Quantity for the 5th, 6th, 7th, *&c.* to the last Inch of the globular Part: See the following

OPERATION.

$Mv = 4$ Inches: Then (by *Prop.* 10) the Measure, in Wine Gallons, of the 1st, 2d, 3d, and 4th, *&c.* Inch from the Top is expressed by

$$\overline{\frac{ab^2}{2} + \frac{cd^2}{2} + .6666} \times .0034.$$

$$\overline{\frac{cd^2}{2} + \frac{ef^2}{2} + .6666} \times .0034.$$

$$\overline{\frac{ef^2}{2} + \frac{gb^2}{2} + .6666} \times .0034.$$

$$\overline{\frac{gb^2}{2} + \frac{ik^2}{2} + .6666} \times .0034.$$

&c,

Hence it appears, that the *accurate* Increases of the 2d Area *(ecdf)* from the 1st, the 3d from the 2d, and the 4th from the 3d Area, *&c.* are respectively equal to the Increases of half the Square of the 3d Diameter *(ef)* from the 1st *(ab)*, the 4th *(gb)* from the 2d *(cd)*, and the 5th *(ik)* from half the Square of the 3d *(ef)*; or, which is the same Thing, the Increases of the 2d Area from the 1st, the 3d from the 2d, the 4th from the 3d, *&c.* are equal to the Decreases of $2Or^2$ from $2OM^2$, $2Os^2$ from $2Om^2$, *&c.* Now, by the preceding *Corollary*, $2OM^2 - 2 \times \overline{OM - 2}|^2$ *less* $2 \times \overline{OM - 1}|^2 - 2 \times \overline{OM - 3}|^2$ is $= 8$ (because n, in this Case, $= 2$, and $\because 2 \times 2n^2 = 8$); consequently the Difference between $2 \times \overline{OM^2 - Or^2}$ and $2 \times \overline{OM^2 - Os^2}$; or (which is still the same) the Difference between $2 \times \overline{rf^2 - Mb^2}$ $(\frac{ef^2}{2} - \frac{ab^2}{2})$ and $2 \times \overline{sb^2 - md^2}$ $(\frac{gb^2}{2} - \frac{cd^2}{2}) = 8$: Which being multiplied by .0034 (in Order to reduce it to Wine Measure), gives .0272, the common Addend, or Subtrahend, according as we begin to inch the Frustum at its greater or less End. Q. E. I.

OPERATION.

Half the Square of 27 the leſs Diameter EF is 364.5

The Square of the Semi-diameter OK (found above) is . . . } 1650.6780
The Square of 37.32 (*Om*) is 1392.7824

Difference 257.8956
Multiply by 2

515.7912
Add 364.5

880.2912
Add .6666

880.9578
Multiplied by .0034

35238312
26428734

Gives the *true* Quantity for the 1ſt Inch from the Top . . . } 2.99525652

The

The Sq. of the Semi-diam. (found above) is . . } 1650.6780
Sub. the Squ. of 36.32 (Or) 1319.1424

Remainder 331.5356

The Double of which is 663.0712
Subtr. ½ the Square of 27 (EF) 364.5

Remainder 298.5712
Multiply by .0034

119428
89571

Reserved Product 1.015138 add to the [1st Inch.
Subtract .0272

Inch.	W. Gallons.
1	2.9952
	Add 1.0151
2	4.0103
	Add .9879
3	4.9982
	Add .9607
4	5.9589
	Add .9335
5	6.8924
	Add .9063
6	7.7987
	Add .8791
7	8.6778
	Add .8519
8	9.5297
	Add .8247
9	10.3544

Reserved Dif. .9879 add to the [2d Inch.
Subtract .0272

.9607 add to the [3d Inch.
Sub. .0272

.9335 add, *&c.*
Sub. .0272

.9063
Sub. .0272

.8791
Sub. .0272

.8519
Sub. .0272

.8247

It

It must be allowed, that, in the preceding Method of gauging a Still, a very small Error may arise, on Account of a little Inclination which is usually given to it, when fixed, as was observed in *Pa.* 126: Nor indeed is there any Method, that I know of, for gauging an inclined Still, but what is liable to some Objection.—For, even supposing we take the Cross-Diameters parallel to the Horizon, and consider the Surface of the Liquor, in any Part of the Body of the Still, to form an Ellipsis (instead of a Circle), we shall then find that the Content of the globular Part of the Still cannot be truly determined by the general Rule given for that Purpose: Besides, the Line PO (see the *Fig. Pa.* 151) will not, in that Case, be the *true Depth* of the Still; for *that* will be represented by the perpendicular Distance of two horizontal Planes, one passing through the highest Point in the Crown, and the other through the lowest Point at the Top of the Still; which Dimension, though differing but *very little* from the Distance PO (*vid. Fig. Pa.* 151), ought to be *truly* known; but *that*, indeed, would be very difficult (if not impracticable) to effect.

Since we cannot determine, where the globular Part of a Still commences; it may be proper to observe, that, in tabulating the whole Content, much Labour will be avoided, if the Altitude of the globular Part be taken a whole Number, and the Decimal Parts (if any happen in the whole Depth) be considered in the Bottom Area: See *Pa.* 153.

Some Authors consider the rising Crown of a Copper or Still, in the Form of the *Segment* of a *Sphere*, and also the Part A*r*O*s*DA (see the *Fig. Pa.* 151) as the Frustum of a *Parabolic Conoid* or *Cone*; and therefore the Quantity of Liquor to cover the Crown will then be determined by the foregoing

foregoing *Prop.* *i. e.* by subtracting the Measure of the Part AODA from that of *ArOs*DA: But, on Account of the Difficulty of obtaining the *true* Diameter and Altitude of the Crown (even admitting the two Figures to be as above represented), I apprehend that *that* Quantity may be found, with much more Certainty and Expedition, by covering (as exact as possible) the highest Point of the Crown with Water, and then carefully drawing off the same, into a Vessel whose Measure is *truly* known.

Note. It very frequently happens the Depth (or Altitude) of a Vessel is such, that the Cross-Diameters, *&c.* cannot all be taken at equal Distances from each other; or, which comes to the same Thing, the said Depth cannot be divided, without a Remainder, by the Number of Areas necessary (and sufficient) to be taken: In that Circumstance, I apprehend, it will be the best Way to consider such Remainder in the uppermost Area, as that Part of the Vessel will be the least subject to cause an Error, in any Charge which may arise from it; not only on Account of the Surface of the Liquor seldom reaching that Area, but because straight-sided Vessels (as *Guile-Tuns*, *Wash-Backs*, *&c.*) generally stand upon their greater Ends. (See *Pa.* 120).

Before I close this *Section*, it may be proper to mention (what was accidentally omitted in *Pa.* 118), that the best Method of obtaining the *true* Depth of a Vessel at the Dipping-place is, with a *Plumb-Line*, thus. — Rub the Bottom of the *Plummet* well with Chalk, *&c.* by which Means, when the *Plumb-Line* is let down from the intended Dipping-place, you will readily discover a Mark made on the Bottom of the Vessel; upon which Mark the End of the *Dimension-Cane* must be placed.

SECTION IX.

OF CASK-GAUGING.

IT has been a general Cuſtom, with Authors on this Subject, to include among the Varieties of Caſks, thoſe of the following Denominations; namely, the Fruſtums of two Parabolic Conoids, and Cones, each of theſe abutting (as it is uſually termed) upon one common Baſe.

But it is well known, from common Experience, that every cloſe Caſk, whether *Pipe*, *Butt*, *Hogſhead*, *&c.* and of what Variety ſoever, is always found to have a Continuity of Curvature at the Bulge, and not to form there an Angle (or ſharp Ridge); which will be actually the Caſe, if we conceive a Caſk to be formed either of two Fruſtums of *Parabolic* (or *Hyperbolic*) *Conoids*, or the Fruſtums of two *Cones:* Therefore, as no ſuch Caſks as theſe are ever made, it cannot, I preſume, be deemed a Crime to expunge thoſe two Varieties; as they have hitherto only embarraſſed the Subject, puzzled the Learner, and even rendered every Perſon, concerned in Caſk-Gauging, more liable to fall into Error.

There is another conſiderable Imperfection in this Branch of Gauging, of which it may be proper to take Notice.

It has been aſſerted by many Authors, who have treated on this Subject, that there is no Rule, or Method, can be given, whereby a Perſon can, with any Degree of Certainty, determine the Variety of the Caſk; that is, whether a Caſk is in the Form

of

of the Middle Fruſtum of a *Spheroid*, *Parabolic Spindle*, or *Hyperbolic Spindle*.

It is true, indeed, no Rules can be given for determining the *exact* Form, or Variety, of the Caſk; but I preſume thoſe which I am going to offer, if duly attended to, will be found of ſingular Uſe, as they will readily diſcover to us, what Variety any Caſk, *very nearly*, approaches to; that is, whether the Caſk may be taken as the Middle Fruſtum of a *Spheroid*, or of a *Parabolic* or a *Hyperbolic Spindle*.

Some Authors direct us to judge from Experience of the Variety of the Caſk: Others divide the Difference between the Spheroidical Caſk, and that compoſed of the Fruſtums of two Cones, into three, or four, equal Parts; and then attempt to lay down Rules for determining theſe different Varieties.

But (even admitting it poſſible that a cloſe Caſk could be formed of the Fruſtums of two Cones) theſe Rules appear to be arbitrary, and to have no Foundation in Science; and likewiſe ſeem to be derived from a Suppoſition that *all* Spheroidical Caſks are the Middle Fruſtums of ſuch Spheroids, whoſe Tranſverſe and Conjugate Axes are in ſome conſtant Proportion; or, which amounts to the ſame, that every Spheroidical Caſk has the ſame Degree of Curvature; but a very ſmall Knowledge in Conic Sections will be ſufficient to convince any One, that there is a vaſt Number of different Forms of Ellipſes, and conſequently Spheroids. — I thought it would be proper to mention this laſt Circumſtance, in Order to rectify an Error which many *practical* Gaugers are apt to fall into, by imagining that a *Spheroidical Caſk* may be known by the Proportion of its Bung and Head-Diameters; or by that of its Bung-Diameter and Length.

Although it may be ſaid, that the following Method is too tedious for ordinary Practice, or for the Officer to aſcertain by it, the Variety of all the different Caſks which daily fall under his Inſpection; yet I dare venture to affirm, that whoever will take the Pains to make themſelves acquainted with the following Directions, will not only be able to diſtinguiſh, *very nearly*, the true Variety of the Caſk; but will, moreover, have a better Idea of it, even by *viewing* the ſame, than by any Method hitherto delivered for that Purpoſe.

The different Forms of Caſks, with Regard to Curvature, may be juſtly comprehended under theſe four Denominations.

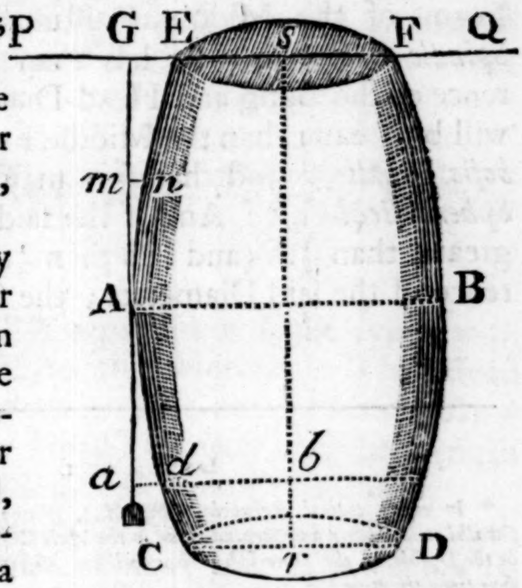
Viz. The Middle Fruſtum of the	*Elliptic Spindle.*	——
	Spheroid	1ſt Variety.
	Parabolic Spindle	2d Variety.
	Hyperbolic Spindle	3d Variety.

But as it very ſeldom (if ever) happens, that a cloſe Caſk is found to contain more than the Middle Fruſtum of a Spheroid; it will therefore be unneceſſary to give any Examples of the Elliptic Spindle: And I have purpoſely omitted the Circular Spindle, on Account of the near Affinity it bears to the Spheroidical Caſk; beſides, the Rule for determining its Content is far too intricate for practical Uſe.

To take the Dimenſions of a ſtanding Caſk; and an expeditious, general, Method of determining, very nearly, the true Variety thereof.

Let AEFBDCA repreſent a Caſk ſtanding upon one Head, with its Axis *(sr)* perpendicular to the Horizon. — Firſt take with your Rule the Diſtance from the Inſide of the Chimb (cloſe to the Head) to the uppermoſt ſlope Edge of the oppoſite

opposite Staff; that Distance will measure the Head-Diameter within the Cask, *very nearly*.



Then lay any straight Rule (or Rod PQ) on the Top of the Cask, passing over the Center (*s*) of the Head, and let a Plumb-Line, with a Noose at one End, be slid backward and forward on the Rod (PQ), till it just touches the Bulge of the Cask at A; measure, carefully, the Distances from the Noose (G) to the out-side of each Chimb at E and F, and from their Sum (*i. e.* the Sum of GF and GE) take twice the Thickness of a Staff at the Bulge of the Cask (as your Judgment directs, according to the Size of the Cask) and the Difference will be the required Bung-Diameter AB.

It is unnecessary to give any Directions for taking the Length of a Cask in this Position, supposing there to be a Hole in the top Head.

But, to determine the Variety of the Cask, proceed thus. — Let $\frac{1}{4}$th of the internal Length of the Cask be set off, from the Bulge (A) towards either Head, on the Plumb-Line (GA*a*), or rather, upon any straight Rod, placed exactly in that Position; that is, let A*m* be equal to $\frac{1}{4}$th of the internal Length of the Cask: Then, if the perpendicular Distance (*mn*) from the Rod to the Cask be equal to $\frac{1}{8}$th of the Difference between the Bung and Head Diameters, the Cask is extremely near (if not exactly) the

Form

Form of the Middle Fruftum of a *Parabolic Spindle*:* But if it be lefs than $\frac{1}{8}$th of the Difference of the Bung and Head-Diameters; the Cafk will be greater than the Middle Fruftum of a *Parabolic Spindle*,† and therefore may be taken as a *Spheroidical Cafk*: And if the faid Diftance *(mn)* is greater than $\frac{1}{8}$th (and lefs than $\frac{1}{4}$th) of the Difference of the faid Diameters; the Cafk, being then lefs

---

LEMMA 1.

* *In every conical Parabola (Fig. X.), if any two Ordinates be drawn parallel to each other and perpendicular to the Axis* CQ, *fo that one* AE *may be the Double of the other* Hb; *then will one Abfciffa* CE *be* exactly *equal to four times the other* Cb.

For, by the Property of the Curve, $Cb : CE :: Hb^2 : AE^2$; but, *by Hypothefis*, $AE = 2Hb$; $\because AE^2 = 4Hb^2$; confequently $Cb : CE$ ($:: Hb^2 : 4Hb^2$) $:: 1 : 4$. Q. E. I.

LEMMA 2.

† *In every Ellipfis (Fig. XI.), if any two Ordinates be drawn parallel to the Tranfverfe Axis, in fuch a Manner, that one* EF *may be the Double of the other* Hb; *then will one Abfciffa* CF *be,* always, *greater than four Times the other* Cb.

Let the Tranfverfe and Conjugate Diameters of any Ellipfis be denoted by *m* and *n* refpectively; alfo let $Cb = x$ and $CF = y$: Then, by the Property of the Curve, we have $\overline{nx - x^2} \times \frac{m^2}{n^2} = Hb^2$, and alfo $\overline{ny - y^2} \times \frac{m^2}{n^2} = EF^2$; but, *by Hypothefis*, $EF = 2Hb$; $\because EF^2 = 4Hb^2$, and therefore $4nx - 4x^2 = ny - y^2$: Hence it is very plain, that if *y* be taken equal to (or lefs than) $4x$, the above Equation is impoffible; for it becomes (by fubftituting $4x$ for *y*) $4nx - 4x^2 = 4nx - 16x^2$, or $x = 2x$, which is *abfurd*: But if, inftead of *y*, dx be wrote in the above general Equation, fuppofing *d* to reprefent any Number (whole or broken) greater than 4; then the faid Equation becomes a poffible one, from whence the Value of *x* (and that of *y*) may be determined. Q. E. I.

Hence it appears, that the above Property obtains in a Circle; that is, if in any Circle, two parallel Chords be fo drawn, that one is the Double of the other; then the Verfed Sine of the greater Segment, will be always more than four Times the Verfed Sine of the lefs: The Truth of which may be, eafily, made out, from a Confideration independent of Algebra.

leſs than a *Parabolic Spindle*, ‡ may be conſidered of the 3d Variety, or the Middle Fruſtum of an *Hyperbolic Spindle*.

To take the Dimenſions, &c. of a lying Caſk.

Let ACDBFEA repreſent a Caſk lying with its Axis parallel to the Horizon: The Head and Bung-Diameters are here obtained in the ſame Manner, as the Head-Diameter and the Length were in the ſtanding Caſk before-mentioned.

The moſt expeditious Way of taking the Length is, with a Pair of Calipers; but as it cannot be expected that every One, concerned in the Art of Gauging, is furniſhed with this Inſtrument; I ſhall therefore lay down the following Method.

Apply any ſtraight Rod (PQ) to the Bulge of the Caſk, in ſuch a Poſition, that a Plumb Line, being

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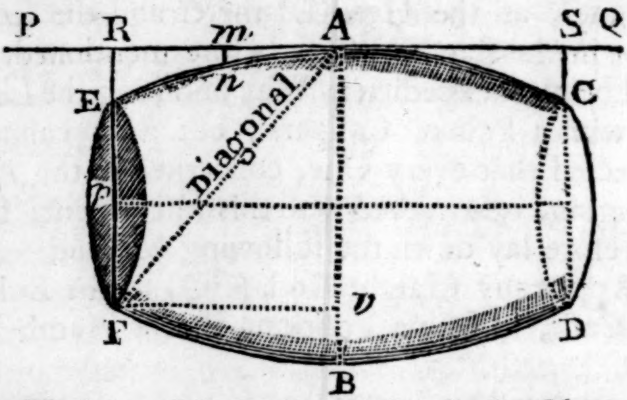
LEMMA 3.

‡ *If two Ordinates (Fig.* XII.) *be drawn in any Hyperbola, perpendicular to the Axis* CQ, *ſo that one* EG *may be the Double of the other* eb; *then will one Abſciſſa* Cb *be, always, greater than* $\frac{1}{4}$*th of the other Abſciſſa* CG, *and leſs than one half thereof.*

Let the Tranſverſe and Conjugate Diameters of any Hyperbola be denoted by m and n reſpectively; alſo let $Cb=x$, and $CG=y$: Then, by the Property of the Curve, we have $\overline{mx+x^2} \times \frac{n^2}{m^2} = eb^2$, and likewiſe $\overline{my+y^2} \times \frac{n^2}{m^2} = EG^2$; but, *by Hypotheſis*, $EG = 2eb$; $\because EG^2 = 4eb^2$, conſequently $my+y^2=4mx+4x^2$: Hence it is evident, if, inſtead of y, there be wrote dx, (for $1 : d :: x : y$), the above general Equation becomes $4mx+4x^2=dmx+d^2x^2$, or $4m+4x=dm+d^2x$, whence $x=\frac{4m-dm}{d^2-4}$; hence the Value of x may be found; provided d repreſents any Number (whole or broken) leſs than 4 and greater than 2; otherwiſe, it is very plain, the Equation would be *abſurd*: Hence alſo the Truth of the *Lemma* is manifeſt. Q. E. I.

being ſuſpended on the Rod, may paſs over (p) the Center of the Head, and obſerve to keep the Rod equally diſtant from each Chimb of the Caſk: This done, and the Rod being kept exactly in this Poſition, lay a ſtraight Rule acroſs each End of the Caſk, to meet the top Rod in R and S; then let the Diſtance RS be carefully meaſured, from which ſubtract the Depths of the Chimbs, together with the Thickneſs of the Heads (as your Judgment directs), the Remainder will be the internal Length of the Caſk.



The Variety of the Caſk may be obtained by meaſuring the perpendicular Diſtance *mn*, and proceeding in the very ſame Manner, as above directed.

Though it is demonſtrable the Property of every Spheroidical Caſk is ſuch, that the Diſtance *mn* (ſee the Figure) may be any Quantity leſs than $\frac{1}{8}$th of the Difference between the Bung and Head Diameters; nevertheleſs, as various Curves may be deſcribed through the ſame three Points, this Property may hold good (with Regard to thoſe Points), and yet the Caſk may, perhaps, be a *ſmall* Matter either greater or leſs, than the Middle Fruſtum of a Spheroid; in which Form it may, however, be always taken, under the above Circumſtance, without ſenſible Error. — The ſame is to be obſerved, with Reſpect to the other two Varieties.

Some

Some Readers may, perhaps, look upon this Method of determining the Varieties of Casks, as a Matter of Speculation only, and not to be regarded in Practice; but, I apprehend, its Utility will so remarkably appear, in the following Examples, particularly in finding the Contents of large Casks, as sufficiently to obviate all Objections on that Head.

EXAMPLE I.

Let it be proposed to find the Content of a Cask in Ale and Wine Gallons; whose Bung-Diameter is 32, Head-Diameter 24, and the Length 42 Inches.

Suppose, by proceeding according to the foregoing Directions, the Distance *mn* (see the last *Fig.*) was found to be something less than one Inch (*i. e.* less than $\frac{1}{8}$th of the Difference of the Bung and Head-Diameters); consequently this Cask, having a Property which appertains to every *Spheroidical Cask*, must be gauged by the following general

RULE.

To twice the Square of the Bung, add once the Square of the Head-Diameter; this Sum being multiplied by the Length, and the Product divided by 1077.15 (*i. e.* three times 359.05) for Ale, or by 882.36 (*i. e.* three times 294.12) for Wine Gallons, will give the Content required.

OPERATION.

Bung-Diameter	32	Head-Diameter	24
	32		24
	64		96
	96		48
	1024	Sq. of the H. Diam.	576
	2		
Twice the Square of the Bung-Diameter	2048		
Add	576		
Sum	2624		
Multiply by the Length	42		
	5248		
	10496		

1077.15)110208(102.31 = the Content in Ale Gallons.

And 110208 being divided by 882.36, the Quotient will be 124.9, the Content in Wine Gallons.

Note. In Practice we may reject the above Decimals in the Divisors, without any material Error in the Result.

EXAMPLE 2.

Wherein it is proposed to find the Content of a Cask in Wine Gallons; whose Bung-Diameter is 65, Head-Diameter 39, and the Length 110 Inches.

Suppose,

Suppose, by the Directions given in *Pa.* 165, the Distance *mn* (see either of the preceding *Figures*) was found to be 3 Inches, which is less than $\frac{1}{8}$th of (26) the Difference of the Diameters; therefore this Cask, having the Property of every *Spheroidical Cask*, must also be gauged by the last *general Rule.*

OPERATION.

Bung-Diameter	65	Head-Diameter	39
	65		39
	325		351
	390		117
The Sq. of the B. Diam.	4225	Sq. of the H. Diam.	1521
The Double is	8450		
Add	1521		
Sum	9971		
Multiply by the Length	110		
	99710		
	9971		

882)1096810(1243.54 = the Content in Wine Gallons.

EXAMPLE 3.

Let it be required to find the Content of a Cask in Ale and Wine Gallons, having the same Length, Bung and Head-Diameters, as that in the 1st Example; only let the Distance *mn* be *now* supposed equal to one Inch.

The Distance *mn* (see the preceding *Fig.*) being *here* equal to $\frac{1}{8}$th of the Difference of the Head and Bung-Diameters; this Cask, therefore, having the very same Property as the Middle Frustum of every *Parabolic Spindle*, must be gauged by the following general

RULE.

To twice the Square of the Bung-Diameter, add the Square of the Head-Diameter, and from this Sum take $\frac{4}{10}$ths (.4) of the Square of the Difference of the said Diameters; multiply the Remainder by the Length of the Cask, and divide the Product by 1077 for Ale, or by 882 for Wine, and the Quotient will give the Content required.

OPERATION.

$$\begin{array}{rr} 32 & 24 \\ 32 & 24 \\ \hline 64 & 96 \\ 96 & 48 \\ \hline 1024 & 576 \\ 2 & \\ \hline \end{array}$$

Twice the Sq. of the B. Diam.	2048
Square of the Head-Diameter	576
Sum	2624
Sq. of 8 the Dif. of the Diam. multiplied by .4	=25.6
Remainder	2598.4
Multiply by the Length	42
	51968
	103936

1077)109132.8(101.32 = the Content in Ale Gallons.

And

And by dividing by 882 (*i. e.* three times 294), the Quotient will be 123.7, the Content in Wine Gallons.

Example 4.

Wherein it is proposed to find the Content of a Cask in Wine Gallons, having the same Dimensions (*i. e.* Bung and Head-Diameters, and Length) as that given in the 2d Example; only suppose the Distance *mn* to measure 3.3 Inches.

In this Case, the perpendicular Distance *mn* (see the preceding *Fig.*) being, *very nearly*, equal to $\frac{1}{8}$th of (26) the Difference of the Bung and Head-Diameters; this Cask must, therefore, be gauged as the Middle Frustum of a *Parabolic Spindle*, by the last-mentioned general Rule.

Operation.

The Sq. of 65, the Bung-Diam. is 4225

The Double of which is 8450
The Sq. of 39, the Head-Diam. is 1521

Sum 9971
The Sq. of 26 the Dif. of Diam. mult. by .4 (*i.e.* 676 mult. by .4) } = 270.4

Remainder 9700.6
Multiply by the Length . . . 110

970060
97006

882)1067066.0(1209.82 W. Gall.

The

The Content of this Cask will, by dividing by 1077, come out equal to 990.77 Ale Gallons.

As no general and practical Rule* can possibly be given for finding, accurately, the Content of a Cask composed of the Middle Frustum of an *Hyperbolic Spindle*, which we here denominate the *3d Variety*; therefore the best Way, when such Casks do occur, will be to have Recourse to the following *general Rule*, which was first, and very judiciously, introduced into the present Subject, by the late ingenious Mr. *Robert Shirtcliffe:* This Method

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* If it was possible to give as *general* and *practical* a Rule, for determining the Measure of the Middle Frustum of an *Hyperbolic Spindle*, as those are for a *Spheroidical Cask*, and *Parabolic Spindle*; the Business of Cask-Gauging might then be justly said to be universally complete; since the Measure of the Frustum of the Hyperbolic Spindle, will be ever some Quantity less than the *Frustum* of a Parabolic Spindle, and greater than *that* of a Cone, the Diameters and Length being supposed the same in each Frustum.

For let A*m*BC*s*DA (*Fig.* XIII.) represent the Frustum of a Parabolic Spindle, and A*n*BC*r*DA that of a Cone; let BE = $\frac{1}{2}$ the Difference of the Diameters, and from F, the Middle of AE, draw the Perpendicular F*m*; also draw *ne* and *mc*, each, parallel to AE: Then (by *Lemma* 1) B*c*=$\frac{1}{4}$BE; but B*e*=$\frac{1}{2}$BE; therefore (by *Lemma* 3) an Ordinate drawn from the Point of Intersection of the Diameter *mn*, and any Hyperbolic Curve, passing through A and the Vertex B, may fall any-where between the Points *c* and *e*; consequently the Hyperbolic Curve may be continually varied, till it becomes coincident either with the Parabolic Curve A*m*B, or the Right-line A*n*B; and that too, without varying the Diameters AD and BC, and the Length AE of the aforesaid Frustums; or, which is the same Thing, the Abscissa BE, and Ordinates AE and *ne*, of the Hyperbolic Curve.

By the very same Method of Reasoning (supposing BG drawn parallel to AE, cutting *nm* produced in *v*) it will appear (by *Lemma* 2) that the Elliptic Curve, passing from A to the Vertex B, cannot descend so *low* as the Point *m*, nor yet rise so *high* as *v*; consequently no close Cask whatever (where the Vertex of the Curve is posited in the Middle of the Cask) can scarce contain more than the Middle Frustum of an *Elliptic Spindle*, nor less than *that* of an *Hyperbolic Spindle*, under the same Dimensions, *i. e.* Head, Bung, and Length.

C O R O L L A R Y.

Hence it appears that two Casks, composed of the Middle Frustums of Hyperbolic Spindles, may have their corresponding Dimensions equal, (*i. e.* the Head, Bung, and Length) and yet the Contents of these Casks may greatly differ: The same is to be observed in the Elliptic Spindle; but this Cask (as before noticed) scarce ever occurs in Practice.

Method evidently follows from that of equidiſtant Ordinates, &c. explained farther on.

Note. I ſhall hereafter exhibit a Table of Multipliers for reducing a Caſk, of this Form of Curvature, into a Cylinder; theſe Multipliers are adapted to a Caſk of ſuch a Degree of Curvature, as I found (from repeated Experiments) would moſt frequently happen in this Variety.

THE GENERAL PROPOSITION.

Having the Length, Bung and Head-Diameters of a Caſk given, and alſo another Diameter taken exactly *in the Middle between the Bung and Head; to find the Content of the Caſk in Ale and Wine Gallons.*

THE GENERAL RULE.

To the Square of the Bung-Diameter, add the Square of the Head-Diameter, and alſo four times the Square of the Diameter taken *exactly* in the Middle between the Bung and Head; the Sum of theſe multiplied by the Length of the Caſk, and the Product divided by 2154.32 for Ale, or by 1764.7 for Wine, the Quotient will be the required Content of the Caſk.*

Note. The Middle-Diameter is eaſily found, by ſubtracting twice the perpendicular Diſtance *mn* (ſee the *Fig. Pa.* 168) from the Bung-Diameter.

EXAMPLE

* Or, univerſally, let the Solid be of what Form ſoever: Add the two extreme Areas, and four times that in the Middle together; multiply the Sum by one-ſixth of the Diſtance of the extreme Areas, and the Product will be the Meaſure of the Solid, *nearly*.

Example 5.

To find the Content of a Cask in Wine Gallons, having the same Bung and Head-Diameters, and Length, as that proposed in the 2d Example; only let the Distance *mn* be *here* supposed equal to 4.55 Inches.

The Distance *mn* (see *Fig. Pa.* 168) being, in this Case, more than $\frac{1}{8}$th of (26) the Difference of the Diameters; this Cask, therefore, having the same Property as that composed of the Middle Frustum of an *Hyperbolic Spindle*, must be gauged by the preceding general Rule.

Operation.

Bung-Diameter 65
Twice the Distance *mn* (4.55) = 9.1

The Diam. in the Middle between the Bung and Head . . . . . } = 55.9

The Square whereof is 3124.81
Multiplied by 4

Gives 12499.24

The Sq. of 65, the B. Diam. is 4225
The Sq. of 39, the H. Diam. is 1521
Add 12499.24

Sum 18245.24
Multiply by the Length . . . 110

18245240
1824524

1764.7)2006976.40(1137.29
= the Content in Wine Gallons.

By

By comparing the preceding Content, with that found for a Caſk of the ſame Dimenſions, in Example the 4th, it will appear that a very conſiderable Error may ariſe from computing the Content of this (or any other) Caſk, by the uſual Methods of only *gueſſing* at its Variety. — It is, indeed, very certain, that this laſt-mentioned *general Rule* will give *very nearly* (and ſometimes *accurately*) the Content of any Caſk, let its Form be what it will; and the nearer the Head-Diameter approaches to an Equality with the Bung-Diameter, the leſs will be the Error: But as we now have a general and practical Method, for diſtinguiſhing the three different Varieties of Caſks; the Contents of the two firſt may, therefore, be found with greater Expedition, by the Rules given for thoſe two Varieties; ſee *Pages* 169 and 172: But, with Reſpect to the 3d Variety, the *general Rule (Pa.* 175) is preferable to all that have hitherto been, or, perhaps, ever can be propoſed.

EXAMPLE 6.

Wherein it is propoſed to find the Content of a Caſk in Wine Gallons; whoſe Bung-Diameter is 31, Head-Diameter 23, Length 50 Inches, and the Diſtance *mn* 1.4 Inches. (See *Fig. Pa.* 165).

The Diſtance *mn* being greater than $\frac{1}{8}$th of (8) the Difference of the Bung and Head-Diameters; this Caſk, therefore, being of the *3d Variety*, muſt be gauged by the laſt-mentioned *general Rule*.

OPERATION:

Bung-Diameter 31
31
———
31
93
———
The Sq. of the B. Diam. 961
The Sq. of 23 the H. Diam. 529
———
Sum 1490

Bung-Diameter 31
Twice 1.4 (the Distance *mn*) = 2.8
———
The Diameter in the Middle between the Bung and Head } 28.2
28.2
———
564
2256
564
———
The Square of 28.2 is 795.24
Multiplied by . . . 4
———
Gives 3180.96
Add 1490.
———
Sum 4670.96
Multiply by the Length . . . 50
———
1764.7)233548.00(132.28
W. Gall.

These Examples, I apprehend, will be sufficient to enable the Learner to find, by the foregoing Method and Rules, the *Variety* and *Content* of a Cask of any other Dimensions.

The

The Contents of Casks may be truly, and more expeditiously obtained, by first finding a *Mean-Diameter*; as will be fully explained in the next Section: But, before we enter upon that, it may not be amiss to give two *general Rules*; one for determining a Diameter taken in the *Middle*, between the Bung and Head-Diameters, of the Spheroidical Cask; and the other for obtaining the same in a Cask composed of the Middle Frustum of the Parabolic Spindle.

PROP. I.

The Bung and Head-Diameters, and the Length of any Spheroidical Cask *being given*; *to find the Diameter* exactly *in the Middle between the Bung and Head.*

RULE.

To three times the Square of the Bung-Diameter, add the Square of the Head-Diameter; $\frac{1}{4}$th of that Sum will be the Square of a Diameter in the *Middle* between the Bung and Head.*

* Let the Bung and Head-Diameters, and $\frac{1}{2}$ the Length of any Spheroidical Cask, be represented by b, h, and d respectively; also let n represent any Distance in the Axis (less than d) from the Bung-Diameter: Then the Square of the Semi-transverse Axis of the whole *Spheroid*, being (by the Property of the Curve) expressed by $\frac{b^2d^2}{b^2-h^2}$, we have (again by the Property of the Curve) $\frac{b^2d^2}{b^2-h^2} : b^2$ (or $\frac{d^2}{b^2-h^2} : 1$) $:: \overline{\frac{bd}{\sqrt{b^2-h^2}}+n} \times \overline{\frac{bd}{\sqrt{b^2-h^2}}-n}$: the Square of the Diameter, at n Distance from the Bung-Circle; which is therefore expressed by $\frac{b^2-h^2}{d^2} \times \overline{\frac{b^2d^2}{b^2-h^2}-n^2}$, or $b^2 - \frac{n^2 \times \overline{b^2-h^2}}{d^2}$: Which, when $n = \frac{d}{2}$ (as in the above *Prop.*), becomes $b^2 - \frac{b^2-h^2}{4}$ ($= \frac{3b^2+h^2}{4}$ = the Square of the Diameter in the *Middle* between those of the Bung and Head. Q. E. I.

PROP. II.

The Bung and Head-Diameters, of the Middle Frustum of a Parabolic Spindle *being given; to find the Diameter in the* Middle *between the Bung and Head.*

RULE.

From the Bung-Diameter, subtract $\frac{1}{4}$th of the Difference of the Bung and Head-Diameters, the Remainder will be the Diameter in the Middle between the Bung and Head.†

EXAMPLE I.

Suppose a *Spheroidical Cask*, whereof the Bung-Diameter is 32, Head-Diameter 24, and the Length 42 Inches; it is required to find the Diameter in the Middle between the Bung and Head: And also the Content of the Cask in Ale Gallons, by the *general Rule, Pa.* 175.

First, by the Rule preceding the last, we have three times the Square of the Bung-Diameter $=3072$

The Square of 24 the Head-Diam. $= 576$

Sum is 3648

$\frac{1}{4}$th is 912, the Square of the Middle-Diameter.

Then, by the *general Rule (Pa.* 175), we have the following

OPERATION.

† This Rule is very evident from the Property of the *Parabola.* (See *Lemma* 1, *Pa.* 166).

OPERATION.

The Sq. of 32 the Bung-Diam. = 1024
The Sq. of 24 the Head-Diam. = 576
Four times 912 the Square of the Middle Diameter } = 3648

Sum 5248
Multiply by the Length . . 42

10496
20992

2154.32)220416(102.32 = the Content in Ale Gallons, *exactly* agreeing with *that* found by the common Method. (See *Example* 1, *Pa.* 169).

EXAMPLE 2.

Wherein it is required to find the *Middle-Diameter*; and also the Content in Ale Gallons (by the *general Rule*, *Pa.* 175) of a Cask of the 2d Variety; whose Bung and Head-Diameters, and Length, are the same as in the preceding Example.

OPERATION.

The Difference of the Diameters 8

$\frac{1}{4}$th is 2, which being subtracted from 32, the Bung-Diameter, leaves 30, the Middle-Diameter; agreeable to the *preceding Rule*.

Then

Then the Sq. of 32, the B. Diam. is 1024
The Square of 24, the H. Diam. is 576
Four times the Square of 30, the Middle-Diameter, is } 3600

Sum 5200
Multiply by the Length . . 42

10400
20800

2154.32)218400(101.37

= the Content in Ale Gallons, *very nearly* agreeing with that found by the common Method, in Example 3d, *Pa.* 171.

The two last Examples were only given *here*, to shew the Conformity between the *general Proposition* (*Pa.* 175.) and the common Method of finding the Contents of these two Casks; which, in the first Variety, will always be an *exact* Agreement;‡ and

‡ The Reason of the *general Rule* (*Pa.* 175) bringing out *precisely* the Measure of the Spheroidical Cask, is from hence.—Let b, b, M, and d, denote the Bung, Head, Middle-Diameter, and Length respectively, of any Spheroidical Cask, and $p = .7854$: Then instead of M^2, in the Expression ($\frac{pd}{6} \times \overline{b^2+4M^2+b^2}$) for the Content, substitute its Equal $\frac{3b^2+b^2}{4}$, found from the Property of the Curve, *Pa.* 179, and we shall get $\frac{pd}{6} \times \overline{b^2+3b^2+b^2+b^2}$ (or $\frac{pd}{3} \times \overline{2b^2+b^2}$); which is well known to be the *accurate* Measure of every Spheroidical Cask.

COROLLARY.

Hence we can easily determine when the Answer brought out by the *general Rule* (*Pa.* 175) is *strictly true*; provided we have another *Rule*, or Method, whereby the *true* Measure of a Plane of a known Form, or a Solid generated by the Revolution of a Curve of a known Property, can be found: For if instead of (M) the Middle Perpendicular (if a Surface), or (M^2) its Square (if a Solid) we substitute its Equal, found from the Property of the Curve (or

and in the second, the Difference will be inconsiderable in Practice.

(or Figure); then, if there results the known *accurate* Rule for determining the Measure of the Figure, it is evident (in such Case) the *general Rule* hereafter given for three equidistant Perpendiculars, and also that given (*Pa.* 175) for three equidistant perpendicular Planes, will be *strictly true*.

Thus, for Instance, in the Frustum of a square Pyramid, if a^2 and b^2 denote the Areas of the two Ends, and d the Altitude of the said Frustum: Then, in the general Expression $\overline{a^2+4M^2+b^2} \times \frac{d}{6}$, for M^2 substitute its Equal $\overline{\frac{a+b}{2}}\Big|^2$, found from the Property of the Figure, and we shall have $\overline{a^2+a^2+2ab+b^2+b^2} \times \frac{d}{6}$, or $\overline{a^2+ab+b^2} \times \frac{d}{3}$; which is known to be the *accurate* Measure of the Frustum. Q. E. I.

OTHERWISE, *let the Frustum of the Pyramid be what it will.*

Let any two homologous Sides of the greater and less similar Ends of the Frustum, be denoted by a and b respectively; and let the corresponding Side of a parallel Section in the Middle be denoted by M: Then will a^2, b^2, and M^2, be as the Measures of the three parallel Planes respectively; and therefore, by the *general Rule*, *Pa.* 175, $\overline{a^2+4M^2+b^2} + \frac{d}{6}$ will be as the Content of the Frustum (d representing the perpendicular Distance of the two extreme Planes): But $\frac{a+b}{2}$ = the corresponding Side of the Plane in the Middle; therefore $\overline{\frac{a+b}{2}}\Big|^2 = M^2$, which being substituted above, in the general Expression for the Content, we get $\overline{a^2+a^2+2ab+b^2+b^2} \times \frac{d}{6}$ or $\overline{a^2+ab+b^2} \times \frac{d}{3}$; which is well known to be as the *accurate* Measure of the Frustum of any Pyramid (or Cone) whatever. (Vid. *Sect.* VIII. *Pa.* 114).

SECTION

SECTION X.

Of finding the Mean-Diameters of Casks.

THERE are two Methods, now in Practice, for finding the Mean-Diameter of a Cask, or reducing it to a Cylinder, of the same Length and Magnitude: The first is, by multiplying the Difference of the Bung and Head-Diameters, by some constant, or fixed, Multiplier (as by .7 for a Spheroidical Cask, .68 for the Middle Frustum of a Parabolic Spindle, *&c.* according to the Variety of the Cask), and adding that Product to the Head-Diameter, this Sum is called the *Mean-Diameter* of the Cask; which is erroneous, as will be shewn hereafter.

The other Method is, by the Tables which are to be found in most Authors on Gauging, and are also graduated on one Edge of the Sliding-Rule; but, it is plain, those Tables are formed from a Consideration that *all* Casks which have the same Difference between the Bung and Head-Diameters, must likewise have one constant Multiplier; therefore this Method is also defective: For it is absolutely impossible there should be any *constant Multiplier*, used in reducing Casks (even of the same Variety) to Cylinders, of the same Lengths and Magnitudes with those Casks; unless it be such as have the Bung and Head-Diameters in some constant Proportion; for the *Multiplier* must vary, when that Proportion varies, as will be hereafter made

made to appear.—It may suffice to shew here, by an easy Example, that two Casks (of the same Variety) may have the very same Difference of their Bung and Head-Diameters, and yet the Proportion of the Diameters of each Cask, may be very different.

Thus, let 32 and 24 Inches be the Bung and Head-Diameters of one Cask, and those of another be 48 and 40 Inches: Here the Difference of the Diameters is the same in each Cask, but the Proportion of their Diameters is unlike; for, in the first Case, the Bung-Diameter contains the Head and $\frac{1}{3}$d Part thereof; but in the latter, the greater exceeds the less by $\frac{1}{5}$th Part *only:* These two Casks therefore, though the Difference of the Diameters is the same in both, require different Multipliers.

From the Investigations* in the subjoined Notes, two different Methods may be given for finding a

 Mean-

* Let the Head-Diameter of any Spheroidical Cask be called x, the Bung-Diameter y, and let m denote some Multiplier, by which the said Bung-Diameter being multiplied, the Product will give the *Mean-Diameter* of the Cask; or, which is the same Thing, the Diameter of a Cylinder, whose Length and Magnitude are equal to *those* of any proposed Spheroidical Cask: Hence (by the well-known Theorem) we get $\frac{2y^2+x^2}{3}=m^2y^2$; therefore $x=\sqrt{3m^2y^2-2y^2}=y\sqrt{3m^2-2}$; consequently the Bung-Diameter is to the Head-Diameter, *universally*, as $y:y\sqrt{3m^2-2}$; or, as $1:\sqrt{3m^2-2}$: From whence it is easy to perceive, that, when the Multiplier (m) is varied, the Ratio of the Bung and Head-Diameters must vary; consequently there cannot be any *constant*, or *fixed*, Multiplier.

Moreover it evidently appears, that the Multiplier (m) cannot be greater than Unity, nor less than $\sqrt{\frac{2}{3}}$; therefore all the Multipliers, or Values of m, let the Bung and Head-Diameters be what they will, are included between 1 and .8164 &c. (*viz.* $\sqrt{\frac{2}{3}}$): But it will be unnecessary to extend any of the Tables, to contain a Multiplier for a Cask (or Vessel) whose Bung (or greater) Diameter, is more than twice the Head (or less) Diameter.

Mean-Diameter, and both of them equally accurate and comprehenfive; the firſt is, by multiplying the Bung-Diameter by a Number, or Factor, according to the Proportion of the Bung and Head-Diameters, this Product will give the *Mean-Diameter:* The other Method is, by multiplying the Difference of the Diameters by a Number, or Factor, which muſt alſo be according to the Proportion of the two Diameters of the Caſk or Veſſel; and that Product being added to the leſs Diameter, the Sum will be the *Mean-Diameter.*

But, as it might be deemed unneceſſary to exemplify both theſe Methods, I thought it would ſuffice to only put down for the firſt, the Tables of Multipliers, as they are derived from a different Conſideration than any hitherto offered to the Public; but for the other Method, I have given both Tables of Factors, and proper Examples to illuſtrate the ſame.

By theſe laſt Tables it will plainly appear, that the common Factors .7, .68, *&c.* uſed in reducing Caſks to Cylinders (notwithſtanding they are better adapted to Practice than any other conſtant Factors whatever), are only *ſtrictly true* in particular Circumſtances: And though the ſaid Factors will be ſufficiently near the Truth, in finding the Contents of many Caſks which occur in Practice; yet, it is very certain, when the Caſk is ſomewhat out of the common Form, the Error will then be far too conſiderable to be diſregarded: So that I preſume theſe Tables will be found of great Utility, in determining the *true* Content of a Veſſel in any of the following Forms; namely, for a cloſe Caſk,

Cask, either in the Form of the Middle Frustum of a *Spheroid*, *Parabolic Spindle*,* or *Hyperbolic* *Spindle*;

* Let the Head-Diameter of a Cask, representing the Middle Frustum of a Parabolic Spindle, be denoted by x, and the Bung-Diameter by y; and let (as in the preceding *Note*) the variable Multiplier be called m: Then (by the Writers on Fluxions) we have $\frac{8y^2+3x^2+4yx}{15}=m^2y^2$, or $\frac{3x^2}{15}+\frac{4yx}{15}=m^2y^2-\frac{8y^2}{15}$; $\therefore x^2+\frac{4yx}{3}=5m^2y^2-\frac{8y^2}{3}$; whence $x=\sqrt{5m^2y^2-\frac{20y^2}{9}}-\frac{2y}{3}=y\times\sqrt{5m^2-\frac{20}{9}}-\frac{2}{3}$; consequently the Bung-Diameter is to the Head-Diameter, *universally*, as $y : y\times\sqrt{5m^2-\frac{20}{9}}-\frac{2}{3}$, or, as $1 : \sqrt{5m^2-\frac{20}{9}}-\frac{2}{3}$: Hence it appears, that, when the Ratio of the Bung and Head-Diameters varies, the Multiplier (m) must vary: Moreover it is evident, that the Multiplier (m) cannot be greater than *Unity*, nor less than $\sqrt{\frac{8}{15}}$, *i. e.* .7302, *&c.*

† Let x be the less, and y the greater Diameter, either of the Frustum of a Parabolic Conoid or that of a Cone; also let m be a Multiplier, by which if the greater Diameter be multiplied, the Product shall be the Mean-Diameter: Hence (by the well-known Theorems) we have the following Equations.

Viz. For the Frustum of a *Parabolic Conoid*, $\frac{y^2+x^2}{2}=m^2y^2$:

And, for the Frustum of a *Cone*, $\frac{y^2+yx+x^2}{3}=m^2y^2$.

These two Equations, being solved, give $x=\begin{cases} y\sqrt{2m^2-1}, & \text{for the Frustum of a } \textit{Parabolic Conoid.} \\ y\sqrt{3m^2-\frac{3}{4}}-\frac{1}{2}y, & \text{for the Frustum of a } \textit{Cone.} \end{cases}$

Therefore the greater Diameter is to the less, *universally*,

As $\begin{cases} 1 : \sqrt{2m^2-1}, & \text{for the Frustum of a } \textit{Parabolic Conoid.} \\ 1 : \sqrt{3m^2-\frac{3}{4}}-\frac{1}{2}, & \text{for the Frustum of a } \textit{Cone.} \end{cases}$

Hence it is evident, that when the Ratio of the two Diameters of each Frustum varies, the Multiplier (m) must vary: It is likewise evident, that the said Multiplier, in the first Case, cannot be greater than Unity, nor less than $\sqrt{\frac{1}{2}}$, or .7071, *&c.* and in the Frustum of a Cone, the Multiplier (m) cannot exceed Unity, nor be less than $\sqrt{\frac{1}{3}}$, or .5773, *&c.*

By

Spindle; ‡ and also, for an open Utensil, the Frustum of a *Parabolic Conoid* and *Cone*.† And

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By the foregoing Proportions it appears, that the Limits of the Multipliers, for the Bung (or greater) Diameters, are

$$\text{Between}\left\{\begin{array}{l} .8164, \&c.\ (\sqrt{\tfrac{2}{3}}) \\ .7302, \&c.\ (\sqrt{\tfrac{8}{15}}) \\ .7071, \&c.\ (\sqrt{\tfrac{1}{2}}) \\ .5773, \&c.\ (\sqrt{\tfrac{1}{3}}) \end{array}\right\}$$

and Unity. — Moreover it is evident (from the Method hereafter shewn of deriving Tables 3 and 4, from 1 and 2 respectively) that the Limits for the Multipliers, for the Difference of the Diameters, are

$$\text{Between}\left\{\begin{array}{ll} .6666, \&c. & (\tfrac{2}{3}) \\ .6666, \&c. & (\tfrac{2}{3}) \\ .5 & (\tfrac{1}{2}) \\ .5 & (\tfrac{1}{2}) \end{array}\right\} \text{ and } \left\{\begin{array}{l} .8164, \&c. \\ .7302, \&c. \\ .7071, \&c. \\ .5773, \&c. \end{array}\right.$$

The *Multipliers* (*Table* I.) for Spheroidical Casks, and for the Middle Frustums of Parabolic Spindles; likewise *those* (*Table* II.) for the Frustums of Parabolic Conoids and Cones, were derived from the *general Proportions* of the Bung and Head, or greater and less Diameters. (See *Pa.* 185 and 187).— For, by assuming the Ratio of the two Diameters, we can readily obtain the Value of m: Thus, for Instance, let the Ratio of the greater and less Diameters be as 2 to 1; then we have

For the Middle Frustum of a *Spheroid*, $1 : \sqrt{3m^2 - 2}$

For the Middle Frustum of a *Parabolic Spindle* . . . . . $1 : \sqrt{5m^2 - \frac{20}{9}} - \frac{2}{3}$

For the Frustum of a *Parabolic Conoid*, $1 : \sqrt{2m^2 - 1}$

For the Frustum of a *Cone* . . . $1 : \sqrt{3m^2 - \frac{3}{4}} - \frac{1}{2}$

$:: 2 : 1$ ($:: 1 : 0.5$).

Whence we get for the

$$\left.\begin{array}{l} 1\text{st} \\ 2\text{d} \\ 3\text{d} \\ 4\text{th} \end{array}\right\} m = \left\{\begin{array}{l} .8660 \\ .8465 \\ .7905 \\ .7637 \end{array}\right\} = \text{the } \textit{general Multipliers} \text{ for the Bung (or greater)}$$

Diameter of any Cask, or Vessel, in the above Forms, whose Diameters are in the Ratio of 2 to 1. — Or if the Bung (or greater) Diameter be expressed by Unity, and the Head (or less) Diameter by $\frac{1}{2}$; then the above Numbers will express the Mean-Diameters themselves.

‡ The Multipliers for the Middle Frustums of the *Hyperbolic Spindle* (*Tab.* I.) were derived in the following Manner.

It was first found (by various Experiments) that many Casks, whose Contents were less than those of *Parabolic Spindles* (having the same Bung, Head, and Length) had the Difference of the Bung and Head-Diameters, and the Distance mn (see *Fig. Pa.* 165 or 168) in the Ratio of 8 : 1.4.

Now if b, h, and M, denote the Bung, Head, and Middle-Diameters respectively of any Cask; and also m the Mean-Diameter thereof: Then, by the

And I flatter myself, that the Advantage of the following Tables will be acknowledged by the *attentive* and *unprejudiced* Reader, as better adapted to *real* Practice, than any hitherto published; considering both the Facility of the Operations, and the Accuracy

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the *general Proposition*, *Pa.* 175, we shall have $\frac{\overline{b^2+4M^2+b^2}}{6}=m^2$, or $m=\sqrt{\frac{b^2+4M^2+b^2}{6}}$: Whence it is evident, that if the Bung-Diameter be denoted by Unity, and the Head-Diameter by any Number less than Unity, suppose, for Example, by .75, we shall have $8 : 1.4 :: .25\ (1-.75) : \frac{1.4\times.25}{8}=.04375$ ($=mn$, see *Fig. Pa.* 168), the Double whereof is .0875: Then $1-.0875=.9125=$ the Middle-Diameter; therefore, in this Case, $b=1$, $b=.75$, and $M=.9125$; consequently $\sqrt{\frac{b^2+4M^2+b^2}{6}}$ $(=m)=\sqrt{\frac{1^2+4\times\overline{.9125}|^2+\overline{.75}|^2}{6}}=.903=$ the Mean-Diameter, or *general Multiplier*, for the Middle Frustum of an *Hyperbolic Spindle* (of this Form) whose Head-Diameter is equal to $\frac{3}{4}$ of the Bung-Diameter.

The 3d and 4th Tables, are respectively deduced from the 1st and 2d; in the following Manner.

Having already proved, that the Multiplier (m) depends intirely upon the Ratio of the two Diameters of the Frustum; and therefore, in the 1st and 2d Tables, the Bung (or greater) Diameter being denoted by Unity, we have the Head (or less) Diameter expressed in Decimal Parts, in the Columns titled *the Quotients of the Head (or less) Diameter, divided by the Bung (or greater) Diameter*; and in the other Columns (titled *Multipliers, &c.*) stand the *true* Mean-Diameters for the respective Cask, &c. whose Bung and Head (or greater and less Diameters) are as here specified: Thus, for Example, call the Bung-Diameter 1, the Head-Diameter .6; then the Mean-Diameter (or Multiplier) of such a *Spheroidical Cask* (by *Tab.* I.) is .887; whence (by multiplying the Difference of the Diameters by x, and adding the Product to the Head-Diameter) we get $.4x+.6=.887$, and $x=\frac{.287}{.4}=.7175$ for a Multiplier; whereby the Difference of the Diameters of every Spheroidical Cask, having the Bung and Head Diameters in the Ratio of 5 to 3, must be multiplied, and the Product added to the Head-Diameter, in Order to obtain the Mean-Diameter.—The same Method must be observed in finding the Multipliers for the other Varieties.

It may not be amiss to take Notice, that when the Quotient, expressing the Value of x terminates at the 3d Decimal, the *Mean-Diameter* of a Vessel, found by *Tables* II. and III. will then *strictly* agree with *that* found by *Tables* I. and II.; but the *Mean-Diameter*, found by the former *Tables*, will differ a *small* Matter, either in Excess or Defect, from *that* found by the latter; according as the Remainder, after the third Decimal is obtained, happens to be greater (equal) or less than $\frac{1}{2}$ the Co-efficient of x.

Accuracy of the Conclusions: For, by the Method here laid down, the Contents of close Casks (and open Vessels) may, with the utmost Exactness, be as expeditiously obtained, as by that ***uncertain*** Method of using the fixed Multipliers.

TABLE I.

Exhibiting the Multipliers, *whereby if the Bung-Diameters of Casks, resembling the Middle Frustums of* Spheroids, *or of* Parabolic *and* Hyperbolic Spindles, *be multiplied, the Products will give the* Mean-Diameters *thereof*; (i. e. *of all such Casks as can be proposed within the Limits of this* Table).

The Quotients of the H. Diam. divided by the B. Diameters.	*The* Multipliers *for Spheroidical Casks.*	*The* Multipliers *for Casks formed of the Frustums of Parabolic Spindles.*	*The* Multipliers *for Casks formed of the Frustums of an Hyperbolic Spindle.*	*The Quotients of the H. Diam. divided by the B. Diameters.*	*The* Multipliers *for Spheroidical Casks.*	*The* Multipliers *for Casks formed of the Frustums of Parabolic Spindles.*	*The* Multipliers *for Casks formed of the Frustums of an Hyperbolic Spindle.*
.50	.8660	.8465	.8136	.76	.9270	.9227	.9068
.51	.8680	.8493	.8170	.77	.9296	.9258	.9105
.52	.8700	.8520	.8204	.78	.9324	.9290	.9143
.53	.8720	.8548	.8239	.79	.9352	.9320	.9181
.54	.8740	.8576	.8273	.80	.9380	.9352	.9219
.55	.8760	.8605	.8308	.81	.9409	.9383	.9257
.56	.8781	.8633	.8343	.82	.9438	.9415	.9295
.57	.8802	.8662	.8378	.83	.9467	.9446	.9333
.58	.8824	.8690	.8413	.84	.9496	.9478	.9372
.59	.8846	.8720	.8448	.85	.9526	.9510	.9410
.60	.8870	.8748	.8483	.86	.9556	.9542	.9449
.61	.8892	.8777	.8518	.87	.9586	.9574	.9487
.62	.8915	.8806	.8554	.88	.9616	.9606	.9526
.63	.8938	.8835	.8590	.89	.9647	.9638	.9565
.64	.8962	.8865	.8626	.90	.9678	.9671	.9604
.65	.8986	.8394	.8662	.91	.9710	.9703	.9643
.66	.9010	.8924	.8698	.92	.9740	.9736	.9682
.67	.9034	.8954	.8735	.93	.9772	.9768	.9722
.68	.9060	.8983	.8772	.94	.9804	.9801	.9761
.69	.9084	.9013	.8808	.95	.9836	.9834	.9801
.70	.9110	.9044	.8845	.96	.9868	.9867	.9841
.71	.9136	.9074	.8881	.97	.9901	.9900	.9880
.72	.9162	.9104	.8918	.98	.9933	.9933	.9920
.73	.9188	.9135	.8955	.99	.9966	.9966	.9960
.74	.9215	.9166	.8993	1.00	1.0000	1.0000	1.0000
.75	.9242	.9196	.9030				

TABLE II.

Shewing the Multipliers, *by which if the greater Diameters of the Frustums of* Parabolic Conoids *and* Cones *be multiplied; the Products will give the* Mean-Diameters *thereof*; (i. e. *of all such Frustums as can be proposed within the Limits of this* Table).

The Quotients of the less Diam. divided by the greater Diam.	*The Multipliers for the Frustums of Parabolic Conoids.*	*The Multipliers for the Frustums of Cones.*	*The Quotients of the less Diam. divided by the greater Diam.*	*The Multipliers for the Frustums of Parabolic Conoids.*	*The Multipliers for the Frustums of Cones.*
.50	.7905	.7637	.76	.8881	.8827
.51	.7937	.7681	.77	.8924	.8874
.52	.7970	.7725	.78	.8967	.8922
.53	.8002	.7769	.79	.9011	.8970
.54	.8036	.7813	.80	.9055	.9018
.55	.8070	.7858	.81	.9100	.9066
.56	.8104	.7902	.82	.9144	.9114
.57	.8140	.7947	.83	.9189	.9163
.58	.8174	.7992	.84	.9234	.9211
.59	.8210	.8037	.85	.9280	.9260
.60	.8246	.8082	.86	.9326	.9308
.61	.8282	.8128	.87	.9372	.9357
.62	.8320	.8173	.88	.9419	.9406
.63	.8357	.8220	.89	.9466	.9455
.64	.8395	.8265	.90	.9513	.9504
.65	.8433	.8311	.91	.9560	.9553
.66	.8472	.8357	.92	.9608	.9602
.67	.8511	.8404	.93	.9656	.9652
.68	.8551	.8450	.94	.9704	.9701
.69	.8590	.8500	.95	.9753	.9751
.70	.8631	.8544	.96	.9802	.9800
.71	.8672	.8590	.97	.9851	.9850
.72	.8713	.8637	.98	.9900	.9900
.73	.8754	.8685	.99	.9950	.9950
.74	.8796	.8732	1.00	1.0000	1.0000
.75	.8838	.8780			

By

By the preceding Tables, the *Mean-Diameters* of Casks, composed of the Middle Frustums of Spheroids, and of Parabolic and Hyperbolic Spindles; and likewise the *Mean-Diameters* of the Frustums of Parabolic Conoids and Cones, may be very readily obtained, by the following general

RULE.

Divide the Head (or less) Diameter, by the Bung (or greater) Diameter, to two Places of Decimals in the Quotient, against which, in the proper Column, we have a *Decimal Fraction*; which being multiplied by the Bung (or greater) Diameter, the Product will give the *true* Mean-Diameter sought.

TABLE

TABLE III.

Exhibiting the Multipliers, *whereby if the Difference of the Bung and Head-Diameters of a* Spheroidical Cask, *or that formed of the Middle Frustum of a* Parabolic *or* Hyperbolic Spindle, *be multiplied, and the Product added to the Head-Diameter; the Sum will give the* Mean-Diameter *thereof*; (i. e. *of any proposed Cask, within the Limits of this* Table).

The Quotients of the H. Diam. divided by the B. Diameters.	The Multipliers for *Spheroidical Casks.*	The Multipliers *for Casks formed of the Frustums of Parabolic Spindles.*	The Multipliers *for Casks formed of the Frustums of an Hyperbolic Spindle.*	*The Quotients of the H. Diam. divided by the B. Diameters.*	The Multipliers *for Spheroidical Casks.*	The Multipliers *for Casks formed of the Frustums of Parabolic Spindles.*	The Multipliers *for Casks formed of the Frustums of an Hyperbolic Spindle.*
.50	.732	.693	.627	.76	.695	.678	.612
.51	.730	.692	.627	.77	.694	.677	.611
.52	.729	.692	.626	.78	.693	.677	.610
.53	.727	.691	.625	.79	.691	.676	.610
.54	.726	.690	.625	.80	.690	.676	609
.55	.724	.690	.624	.81	.689	.675	.609
.56	.723	.689	.623	.82	.688	.675	.608
.57	.721	.689	.623	.83	.686	.674	.608
.58	.720	.688	.622	.84	.685	.674	.607
.59	.719	.688	.621	.85	.684	.673	.607
.60	.717	.687	.621	.86	.683	.673	.606
.61	.716	.686	.620	.87	.682	.672	.605
.62	.714	.686	.619	.88	.680	.671	.605
.63	.713	.685	.619	.89	.679	.671	.605
.64	.712	.685	.618	.90	.678	.671	.604
.65	.710	.684	.618	.91	.677	.670	.603
.66	.709	.684	.617	.92	.675	.670	.603
.67	.708	.683	.617	.93	.674	.669	.603
.68	.706	.682	.616	.94	.673	.668	.602
.69	.705	.682	.615	.95	.672	.668	.602
.70	.703	.681	.615	.96	.670	.667	.602
.71	.702	.681	.614	.97	.670	667	.601
.72	.701	.680	.614	.98	.667	.666	.601
.73	.699	.680	.613	.99	.666	.666	.600
.74	.698	.679	.613	1.00	—	—	—
.75	.697	.678	.612				

Note. *The above Table (for the Sake of Convenience) is now graduated on the* Sliding-Rule, *as made*

made by those ingenious Mathematical Instrument-Makers, Mr. John Bennett, *in* Crown-Court, St. Ann's, Soho; *and Mr.* Edward Roberts, *in* Dove-Court, Old Jewry.

Although two Places of Decimals being taken for a Multiplier, in the Manner as they are now placed on the Sliding-Rule by the above-mentioned Gentlemen, will give the Mean-Diameter of a Cask to a surprizing Degree of Exactness; yet I judged it would not be amiss to give three Places in the preceding Table, in Order to shew in what Circumstances (with Regard to the Proportion of the Bung and Head-Diameters) the common Multipliers (.7 and .68) will be the most exact.

Before I proceed to shew the Utility of this last Table, by the Application of the following *general Rule*, it may be proper to observe, that it will require a *Sliding-Rule* of 18 Inches long to graduate thereon, distinctly, the preceding Table, and to elucidate the same with three Places of Decimals: To effect which, in the most concise Manner, take the following Directions.

Mark down the Quotients of the Head-Diameter divided by the Bung-Diameter in one Line; thus, .50, .51, .52, .53, *&c.* then directly under .50 place, successively one under another, .732, .693, and .627; being the Multipliers for the 1st, 2d, and 3d Variety respectively: Under .51, successively also one under another, place only 0, 2, and 7; being the third Decimal of the Multipliers .730, .622, and .127 respectively: Again, under .52 place .729, 1 and 6. Proceed in this Manner, placing only the third Decimal of the Multiplier (as it appears in the Table)

Table) under the proper Quotient of the Head-Diameter divided by the Bung-Diameter; except when the ſecond Decimal of the Multiplier is diminiſhed; in which Caſe it will be neceſſary to put down the whole Factor (agreeable to the Table), inſtead of only the laſt Decimal of it.

A Sliding-Rule of 12 Inches long will be ſufficient to contain ſo much of the ſaid Table as generally occurs in Practice; namely, the Quotient of the Head-Diameter divided by the Bung-Diameter from .60 to .96: But, if the Multipliers are only to be graduated to two Places of Decimals, the greateſt Extent of the preceding Table may be clearly explained on a Foot-Rule; provided the Quotient of the Head-Diameter divided by the Bung-Diameter is marked down in the following Manner; *viz.* .50, 1, 2, 3, 4, 5, 6, 7, 8, 9. .60, 1, 2, 3, 4, 5, *&c.* to 1.0: For it appears from the preceding Table, that, if the Quotient of the Head-Diameter divided by the Bung-Diameter is not greater than .54, the neareſt Multiplier, to two Places of Decimals, is .73 for a Spheroidical Caſk; if the ſaid Quotient is not greater than .64, the Multiplier, for the Fruſtum of a Parabolic Spindle, muſt be .69; and, if the ſaid Quotient does not exceed .54, the Multiplier, for the Fruſtum of an Hyperbolic Spindle, will be .63: Moreover it appears from the foregoing Table, that, if the Quotient of the Head-Diameter divided by the Bung-Diameter, be greater than .54, and leſs than .62, the Multiplier for a Spheroidical Caſk, muſt be .72; which place, on the Rule, oppoſite .58, being the Middle between the two laſt-mentioned Quotients .54 and .62: Proceed in the ſame Manner with the reſt of the Table.

GENERAL RULE.

Divide the Head-Diameter by the Bung-Diameter, to two Places of Decimals in the Quotient, against which, in the Column answering to the proposed Variety, we have a Decimal; which being multiplied by the Difference of the Bung and Head-Diameters, and the Product being added to the Head-Diameter, the Sum thereof will be the *true* Mean-Diameter sought.

EXAMPLE I.

Wherein it is proposed to find the Mean-Diameter, and Content of a *Spheroidical Cask* in Wine Gallons; whereof the Bung-Diameter is 65, Head-Diameter 39, and the Length 110 Inches.

OPERATION.

65) 39.0(.6 *Quotient.*
390

. . .

Then

Then against .60 (*Tab.* III.) in the first Column, and in that for Spheroidical Casks, we have the Multiplier (or Factor) . . . = .717
Multiplied by the Diff. of the Diam. = 26

	4302
	1434
Product	18.642
Head-Diameter	39
Mean-Diameter	57.642

: The Area in Wine Gallons, answering to this Diameter, is . . . . 11.302, *very nearly.*

Multiplied by the Length	110
	113020
	11302
Gives	1243.220

Wine Gallons, the Content of the Cask; the same as was found in *Example* 2, *Pa.* 171.

If, in the foregoing Example, the Difference of the Bung and Head-Diameters be multiplied by .6 (agreeable to an Observation of a very celebrated Author), the Mean-Diameter will come out 54.6 Inches; and therefore the Content of the Cask will then *appear* to be but 1114.3 Wine Gallons, which is 129 Gallons *less* than the *Truth!*

EXAMPLE 2.

Let it be proposed to find the Mean-Diameter, and Content in Wine-Gallons, of a Cask representing the Middle Frustum of a *Parabolic Splindle*; whose Bung-Diameter is 32, Head-Diameter 24, and the Length 42 Inches.

OPERATION.

OPERATION.

32)24.00(.75 *Quotient*

Then against .75 (*Tab.* III.), in the proper Column for this *Variety*, we have .678 for a Multiplier; and therefore, by proceeding as in the last Example, the Mean-Diameter is 29.424, and the required Content 123.648 Wine Gallons; the same as found by the general Rule, *Exam.* 3, *Pa.* 172.

By the Sliding-Rule.

To the Wine Gauge-point on D, set the Length 42 on C; then against 29.4 the Mean-Diameter on D, we have 124 Gallons *nearly*, the Content of the Cask on C.

EXAMPLE 3.

Suppose the Dimensions of a Cask of the 3d Variety (or the Middle Frustum of an *Hyperbolic Spindle)* be the same as were given in Example 2, *Pa.* 170; to find the Mean-Diameter of the Cask, and its Content in Wine Gallons.

OPERATION.

Operation.

If the Head-Diameter 39, be divided by the Bung-Diameter 65, the Quotient will be .6; against which (*Tab.* III.), in the Column proper for this Variety, we have . . .621

Multiplied by the Difference of the Bung and Head-Diameters	26
	3726
	1242
Product	16.146
Head-Diameter	39

Mean-Diameter is 55.146: The Area in Wine Gallons, answering to this Diameter, is 10.34, which being multiplied by the Length (110) gives 1137.4 Wine Gallons, the required Content of the Cask: Which differs 106 Gallons from one of a spheroidical Form, having the same Bung, Head, and Length, see *Pa.* 171; but agrees, *very nearly*, with the Content found according to the *general Rule*, *Example* 5, *Pa.* 176.

By the Sliding-Rule.

To the Wine Gauge-point on D, set the Length 110 on the Line C (*i. e.* on the 1st Radius); then against the Mean-Diameter 55.14 on D, we have 1137.4 Gallons, the Content on C, *as before.*

Example 4.

Wherein it is proposed to find the Mean-Diameter, and Content of a Cask of the 3d Variety (or the Middle

Middle Fruſtum of an *Hyperbolic Spindle)*, whoſe Bung-Diameter is 31, Head-Diameter 23, and the Length 50 Inches.

By dividing the Head by the Bung-Diameter, and proceeding in the very ſame Manner as in the foregoing Examples, we ſhall find the Mean-Diameter 27.89, and the Content of the Caſk 132.25 Wine-Gallons; the ſame as in *Pa.* 178.

By the Sliding-Rule.

To the Wine Gauge-point on D, ſet the Length 50 on C; then againſt 27.9 the Mean-Diameter on D, we have 132.25 Wine Gallons, the Content on C; the ſame *as above.*

Note. If the above Example be wrought by the common Method, of uſing .7 for a Multiplier, the Content will then *appear* to be 139 Gallons, which exceeds the *true* Meaſure 6.75 Gallons.

TABLE

TABLE IV.

Shewing the Multipliers, *whereby if the Difference of the Diameters of the Frustums of any* Parabolic Conoid *or* Cone *be multiplied, and the Product added to the less Diameter; the Sum will give the* Mean-Diameters *thereof*; (i. e. *of all such Frustums as can be proposed within the Limits of this* Table).

The Quotients of the less Diam. divided by the greater Diameters.	*The Multipliers for the Frustums of Parabolic Conoids.*	The Multipliers for the *Frustums of Cones.*	*The Quotients of the less Diam. divided by the greater Diameters.*	*The Multipliers for the Frustums of Parabolic Conoids.*	The Multipliers for the *Frustums of Cones.*
.50	.581	.527	.76	.534	.511
.51	.579	.527	.77	.532	.510
.52	.577	.526	.78	.530	.510
.53	.575	.526	.79	.529	.510
.54	.573	.525	.80	.527	.509
.55	.571	.524	.81	.526	.508
.56	.569	.523	.82	.524	.508
.57	.567	.523	.83	.522	.508
.58	.565	.522	.84	.521	.507
.59	.563	.521	.85	.520	.506
.60	.562	.521	.86	.519	.506
.61	.559	.520	.87	.517	.505
.62	.558	.519	.88	.516	.505
.63	.556	.519	.89	.515	.504
.64	.554	.518	.90	.513	.504
.65	.552	.517	.91	.511	.503
.66	.551	.517	.92	.510	.503
.67	.549	.516	.93	.509	.503
.68	.547	.516	.94	.507	.502
.69	.545	.516	.95	.506	.501
.70	.543	.515	.96	.505	.500
.71	.541	.514	.97	.503	.500
.72	.540	.513	.98	.501	.500
.73	.539	.513	.99	.500	.500
.74	.537	.512	1.00	—	—
.75	.535	.512			

It appears from the foregoing Table, that the Mean-Diameter of the Fruſtum of a Cone is *nearly* equal to half the Sum of the top and bottom Diameters of the ſaid Fruſtum; eſpecially when the leſs Diameter is more than $\frac{2}{3}$ds of the greater: But this Obſervation, it is evident from Table III. will not hold good, with Reſpect to the other Fruſtums, in any Circumſtance whatever.

EXAMPLE 5.

The greater Diameter of the Fruſtum of a *Parabolic Conoid* is 45, the leſs Diameter 27, and the Altitude 40 Inches; required the Mean-Diameter of the Fruſtum, and its Content in Wine Gallons.

OPERATION.

45)27.0(.6 *Quotient.*

Then againſt .60 (*Tab.* IV.), in the Column for the Fruſtums of *Parabolic Conoids*, we have .562, which being multiplied by the Difference of the Diameters (18), gives 10.116; to which add the leſs Diameters (27), and the Sum will be the required Mean-Diameter 37.116. — The Area in Wine Gallons, anſwering to this Diameter, is 4.683, which being multiplied by the Altitude 40

Gives 187.320 Wine Gallons, the Content ſought; *very nearly* the ſame as in the *Example*, *Pa.* 150.

By the Sliding-Rule.

To the Wine Gauge-point on D, ſet the Altitude (or Length) 40 on C; then oppoſite 37.1 the above Mean-Diameter on D, we have 187.3 Wine Gallons, the Content on C.

EXAMPLE

EXAMPLE 6.

Let the leſs Diameter of the Fruſtum of a *Cone* be 22, the greater Diameter 40, and the Altitude 60 Inches; required the Mean-Diameter and Content of the Fruſtum in Wine Gallons.

OPERATION:

40)22.00(.55 *Quotient.*

Then againſt .55 (*Tab.* IV. *Pa.* 201.) in the Column for the Fruſtums of Cones, we have for a Multiplier .524; whence, by proceeding as in the laſt Example, the Mean-Diameter comes out 31.432, and therefore the Content is 201.48 Wine Gallons. (See *Pa.* 116).

By the Sliding-Rule.

To the Wine Gauge point on D, ſet the Altitude 60 on C; then againſt 31.43, the Mean-Diameter on D, we have 201.5 Gallons, the Content on C, *as before.*

The Mean-Diameters, found in the preceding Examples, may be alſo obtained by the 1ſt and 2d *Tables.* — Thus, againſt the laſt-mentioned Quotient .55 (*Tab.* II.), we have for the Fruſtum of a Cone 7858
Multiplied by the greater Diam. 40

Gives the Mean-Diameter 31.4320, the ſame as above: Which, in this Caſe, is found with more Expedition than by the other Method; but it muſt be obſerved, that the ſaid Method is very often more conciſe than by the 1ſt and 2d Tables, by Reaſon that two Figures (and in ſome particular

Circumſtances *one)* taken for a Multiplier in the 3d and 4th Tables, will be as exact, as when four Figures are taken for a Multiplier in the 1ſt and 2d Tables.

It may be proper to mention here, that if the exact *Quotient* of the Head (or leſs) Diameter divided by the Bung (or greater) Diameter cannot be found in the preceding *Tables*; then will the Mean-Diameter differ a ſmall Matter from the Truth; but, however, the greateſt difference that can ever happen by the *Tables*, will be wholly inconſiderable in Practice; and that Difference will even become leſs, if we obſerve to take out the Multiplier which anſwers the neareſt to the ſaid *Quotient*. Thus, for Inſtance, let the Head-Diameter be 21.7, and the Bung-Diameter 32 Inches; theſe being divided as above-mentioned, the *Quotient* will be .678, *&c.* therefore the *Number* againſt .68, in the preceding *Tables*, will be more exact for a *Multiplier* than *that* againſt .67.

Of the Conſtruction and Properties of the Diagonal Rod.

(See *Fig. Pa.* 168).

The Diviſions graduated on this Inſtrument are founded upon the well-known Property of ſimilar Solids; namely, that their Contents are to one another as the Cubes of their homologous (or like) Sides, or Dimenſions.

If the Bung-Diameters, Head-Diameters, and Lengths of any two Caſks (of the ſame Variety) are in the ſame Proportion to each other, thoſe Caſks

Casks are *then* alike in Form, or similar; and their Contents will be to each other, as the Cubes of their Corresponding Dimensions, and therefore (in this Case) as the Cubes of their Diagonals. Hence it appears, that the original Construction of the Diagonal Line was extremely easy: For the Bung and Head-Diameters, Length and Variety, of such a Cask as best agreed with the most general Form of Casks,* being first carefully taken in Inches

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* It is utterly impossible to investigate what particular Form of a Cask was first fixed upon, in the original Construction of the Diagonal Line; even supposing the Property of the Curve of the Cask known: It must be allowed that there is an infinite Number of different *Forms* and *Magnitudes* of each Variety of Casks which have the very same Diagonal; nay, even in every close Cask but a cylindrical One, both the Diagonal and Content thereof may remain the same, and yet the Form of it, or the Ratio of the Head and Bung-Diameters, and Length, may vary; because it is evident that the Diagonal and *one* other Dimension, being known, are not sufficient to *limit*, neither the *Figure*, nor *Magnitude* of the Cask; whence it is plain, that, besides the Diagonal, there must be given another Dimension, in Order to obtain the Form of a Cask of a *given* Magnitude; which Magnitue, in the present Case, is always a given *(fixed)* Multiple of the Cube of the Diagonal: Therefore, if the Bung, or the Head, or the Ratio of the Bung and Head-Diameters is known; then, with any of these, and a given Diagonal and Content answering thereto, the Form of the Cask (supposing the Nature of the Curve known) will be easily determined; provided the greatest Content that can be formed with the above *Data* is equal to, or exceeds the Content answering to the proposed Diagonal, or, which comes to the same Thing, to a given Multiple of its Cube; which Circumstance is known to obtain, in a Spheroidical Cask, in *every* Ratio of the Bung and Head-Diameters within a certain Limit; namely, when the Head-Diameter does not exceed nine-tenths of the Bung-Diameter: As will manifestly appear from the following Question, which I published in the *Ladies Diary*, 1763; and also its Solution in the subsequent Diary: But the Limit for the 3d Variety is (Hereafter) found to be, when the Head-Diameter does not exceed eighty-three hundredths of the Bung-Diameter.

Question. — Let the Ratio of the Head and Bung-Diameters be what it will, within certain Limits, a Spheroidical Cask may be so formed, that the Diagonal Line such as is now graduated upon Gauging-Rods, will exhibit the true Content of the Cask: 'Tis proposed to find, by a general Method, what those Limits are, and how near the Head and Bung-Diameters can approach to the Ratio of Equality, before the above Circumstance fails.

Solution.

Inches and Tenths: Then the Square (Fv) of half the Length of the Cask (see *Fig. Pa.* 168.) being

SOLUTION. — Let the given Diagonal of the required Spheroidical Cask be denoted by d, its Bung-Diameter by x; also let the Ratio of its Head and Bung Diameters be as n to 1, and .78539, *&c.* the Area of a Circle whose Diameter is Unity, $= p$: Then, by the well-known Theorem, the Content thereof will be expressed by $\frac{2p}{3}\sqrt{d^2 - \overline{\frac{1+n}{2}}\big|^2 \times x^2} \times \overline{2x^2 + n^2x^2}$; which, put into Fluxions (supposing n constant) and made $= 0$, and reduced, gives $x = \frac{2d\sqrt{2}}{\overline{1+n} \times \sqrt{3}} =$ the Bung-Diameter, in this Circumstance, when the Cask, under the Diagonal d, is the greatest possible; and this Value, substituted for x, in the above Expression, gives $\frac{16pd^3}{9\sqrt{3}} \times \frac{2+n^2}{\overline{1+n}\big|^2} =$ the Content of the *greatest Spheroidical Cask* having the given Diagonal d, and corresponding to any assignable possible Value of n: And when it is less than $d^3 \times .6283$ *(nearly)*, the Content (in Inches) as found by the Diagonal Line, the *Prob.* is, then, manifestly impossible; suppose *it*, therefore, equal thereto, in Order to determine the Limit of n (required); *i. e.* suppose $\frac{16pd^3}{9\sqrt{3}} \times \frac{2+n^2}{\overline{1+n}\big|^2} = d^3 \times .6283$, *&c.* or $\frac{2+n^2}{\overline{1+n}\big|^2} = .7796$; whence n is readily found $= .898$, *&c.* — Hence it appears, that the *present-constructed* Diagonal Line will not exhibit the true Content of a Spheroidical Cask, when the Ratio of the Head and Bung-Diameters approaches nearer to an Equality than that of .898 to 1 (or *nearly* 9 to 10), in any Circumstance whatever.

COROLLARY I.

In the above Equation $\left(\frac{2+n^2}{\overline{1+n}\big|^2} = .7796\right)$ n has two Values; *i. e.* 6.16, *&c.* .898, *&c.* but the former of them can, evidently, have no Place in the preceding Question; because the Curvature of every close Cask must be, always, concave to the Axis thereof, and consequently the Bung-Diameter always greater than the Head-Diameter. — It may be farther necessary to observe, that within the Limits of the two Roots .898, *&c.* and 6.16, *&c.* the greatest Content of a Spheroidical Cask which can be formed, with a given Diagonal, will be less than is shewn on the Rod by that Diagonal; and will be the least possible when $n = 2$, and the greatest when $n = 0$.

COROLLARY.

being added to the Square (Av) of half the Sum of the Bung and Head-Diameters; and against the

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COROLLARY 2.

Let every Thing be interpreted as in the preceding Solution: Then, by the Method of forming the Table of the third Variety, *Pa.* 188, we have $8 : 1.4 :: x - nx\ (\overline{1-n} \times x) : \frac{1.4}{8} \times \overline{1-n} \times x$, or $\overline{.175-.175n} \times x = mn$ (see *Fig. Pa.* 168); therefore $\overline{.35-.35n} \times x = 2mn$, and consequently $x - .35x + .35nx$, or $.65x + .35nx = M$; $\therefore \overline{.65x+.35nx}|^2 (= .4225x^2 + .455nx^2 + .1225n^2x^2) = M^2$: Whence, by the general Theorem, *Pa.* 183, $\frac{x^2 + .4225x^2 + .455nx^2 + .1225n^2x^2 \times 4 + n^2x^2}{6} \times 2p\sqrt{d^2 - \frac{\overline{1+n}|^2}{2}} \times x^2$ will express the Content of the Cask in Inches; which being put into Fluxions, (supposing n constant, *as before*) and made $= 0$, and reduced, we get $x^2 = \frac{2d^2}{\frac{\overline{1+n}|^2}{2} \times 3}$; this being substituted for x^2, in the above general Equation, we then get $\frac{9p}{6} \times$

$$\sqrt{d^2 - \frac{\overline{1+n}|^2}{2} \times \frac{2d^2}{\frac{\overline{1+n}|^2}{2} \times 3}} \times \overline{2.69+1.82n+1.49n^2} \times \frac{2d^2}{\frac{\overline{1+n}|^2}{2} \times 3} = d^3 \times .6283, \text{ or } \frac{16p}{18\sqrt{3}} \times \frac{2.69+1.82n+1.49n^2}{\overline{1+n}|^2} =$$

$.6283$, or $n^2 + \frac{1.2984}{.0692} \times n = \frac{1.1308}{.0692} = 16.34$, &c. whence $n = .83$, *very nearly*. — Hence it appears, that if the Head and Bung-Diameters of a Cask of the 3d *Variety*, approach nearer to the Ratio of Equality than .83 to 1, the Diagonal Line, such as is now constructed on Gauging-Rods, will fail to exhibit its true Content, let the Length of the Cask be what it will.

Now, in Order to determine which is the most general Form of Spheroidical Casks, to be met with in Practice, whose Contents will be truly obtained by the common Diagonal Line, we must assume, to any given Content, either the Bung, or Head, or Ratio of the Bung and Head-Diameters (such as is known to occur, according to the most general Form of Casks); then the Question becomes limited, and the other Dimensions of the Cask may be found, so that the Diagonal Line will exhibit its true Content.

Let

the Square Root of that Sum, measured on a ſtraight Rod (or Rule), were placed the true Contents

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Let, for Inſtance, the Content of a Spheroidical Caſk be $110\frac{1}{2}$ Wine Gallons (its Diagonal, on the Rod, being *nearly* 34.4 Inches), and let the Ratio of the Bung and Head-Diameters be as 1 to .85; moreover let the Bung-Diameter be denoted by x, and $p = .7854$: Then, by *Tab.* I. *Pa.* 190, we have .9526 for a Multiplier, therefore $.9526x$ will expreſs the Mean-Diameter of the required Caſk; but $.925x = \frac{1}{2}$ the Sum of the Bung and Head-Diameters; conſequently the Length of the Caſk will be expreſſed by $2\sqrt{1183.36 - \overline{.925x}|^2}$; whence $2\sqrt{1183.36 - .8556x^2} \times \overline{.9526x}|^2 \times p = 25525.5$, and $\because x = 32$, the required Bung-Diameter, *nearly*; conſequently the Head-Diameter $(= 32 \times .85) = 27.2$, and the Length $(= 2\sqrt{1183.36 - .8556x^2}) = 35$, *nearly*.

It is very evident that this Method is applicable, in like Manner, to any of the other Varieties, as well as the Spheroidical Form. — Suppoſe, for Inſtance, the Content of a Caſk of the 3*d Variety* to be $132\frac{3}{4}$ Wine Gallons, whereof the Diagonal (as near as can be determined by Inſpection) on the Rod, is 36.52 Inches (or $\sqrt[3]{\frac{132.75}{.00272}}$, *nearly*), and alſo let the Ratio of the Bung and Head-Diameters be as 1 to .8: Then, by *Tab.* I. *Pa.* 190, against .80 we have .9219 for a Multiplier; therefore $2 \times \sqrt{\overline{36.52}|^2 - \overline{\frac{1.8x}{2}}|^2}$ will expreſs the Length of the Caſk, and conſequently $2\sqrt{\overline{36.52}|^2 - .81x^2} \times \overline{.9219x}|^2 \times p = 132.75 \times 231 = 30665.25$, or $2\sqrt{1333.7104 - .81x^2} \times .8499x^2 \times .7854 = 30665.25$; whence $x = 32 =$ the Bung-Diameter, and therefore $32 \times .8 = 25.6 =$ the Head-Diameter, and the Length $(= 2\sqrt{1333.7104 - .81x^2}) = 44.91$ Inches.

OTHERWISE, *without conſidering the Magnitude of the Caſk.*

If the Bung and Head-Diameters, and Variety of a Caſk are known, we can readily determine the Length thereof, ſo that the Diagonal Line ſhall exhibit the true Content of ſuch Caſk.

Let the Bung and Head-Diameters be repreſented by a and b reſpectively, $r = .00272$ (= the common Multiple of the Cube of the Diagonal, for Wine Gallons, *nearly*); alſo let A = the Area of a Circle in Wine Gallons, whoſe Diameter is = the Mean-Diameter of the Caſk, and $x = \frac{1}{2}$ the required Length thereof: Then will $\sqrt{\overline{\frac{a+b}{2}}|^2 + x^2} =$ the Diagonal; and therefore

Contents of the Cask in Ale and Wine Gallons, found (by the general Method) agreeable to the E e Variety

therefore $\overline{\frac{a+b}{2}\Big|^2 + x^2} \times \sqrt{\overline{\frac{a+b}{2}\Big|^2 + x^2}} \times r = A \times 2x$. Now let, for Example, $a = 32$, $b = 25.6$; and also let the Cask be of the 3d Variety: Then, by *Tab.* I. *Pa.* 190, against .80 (or $\frac{25.6}{32}$) we have .9219 for a Multiplier, whence the Mean-Diameter is $= 29.5$ ($= .9219 \times 32$), and therefore $A = 2.9588$; consequently the above *general Equation*, in Numbers, becomes $\overline{829.44 + x^2} \times \sqrt{829.44 + x^2} \times .00272 = 2.9588 \times 2x$; whence $x = 22.455$, and ∴ the required Length ($2x$) is 44.91; *the very same as above.*

COROLLARY.

It appears, from Sir ISAAC NEWTON's Method of determining the Roots of Equations, that the last general Equation contains four *impossible Roots*, and that the other two will be real *affirmative Ones*: This Circumstance holds good in every Ratio of the Bung and Head-Diameters; except when the said Ratio approaches so near to that of Equality, as 1 to .898, in a Spheroidical Cask, or as 1 to .83 in a Cask of the 3d Variety. Whence it is plain, that, as the *Ratio* of the Bung and Head-Diameters approaches nearer and nearer to *those* above-mentioned, the Limits of the said *affirmative Roots* become narrower and narrower, till they (at last) coincide in the said Ratios.—In the last Example, or the general Equation in Numbers, x has two affirmative Values; *i. e.* 22.455 and 18.55; whence it appears, that if the Head-Diameter, Bung-Diameter, and Length of a Cask, of the 3d Variety, are respectively equal to, or in the Proportion of, 25.6, 32, and 44.91; or 25.6, 32, and 37.1; the Content thereof will be truly exhibited by the common Diagonal Line.

There are other general Methods for determining the *Figure* of a Spheroidical Cask (and also, if it was necessary, *that* of a Parabolic Spindle), whose Content will be truly obtained by the Diagonal Line. — The following Investigation is on a Supposition, that the Content of the Cask, its corresponding Diagonal (on the Rod), and the Ratio of the Bung and Head-Diameters are known.

Let the Content of a Cask, in cubic Inches, be denoted by a, its Diagonal (on the Rod) by d, and the required Semi-length by x; also let the given Ratio of the Head and Bung-Diameters be as n to 1, ($p = .7854$): Then $\sqrt{d^2 - x^2} = \frac{1}{2}$ the Sum of the Bung and Head-Diameters, and ∴ $\frac{2\sqrt{d^2 - x^2}}{n+1}$ = the Bung-Diameter, (for $n : 1 ::$ Head : Bung, and, by Composition, $n + 1 : 1 ::$ H + B : B), and $\frac{2n\sqrt{d^2 - x^2}}{n+1}$ = the

Head-

Variety thereof: Then, by the Nature of similar Solids, it will, universally, hold, as the Cube of

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Head-Diameter; whence we have $\frac{8 \times \overline{d^2-x^2} + 4n^2 \times \overline{d^2-x^2}}{\overline{n+1}|^2} \times \frac{2px}{3} = a$, for a *Spheroidical Cask.* (And we likewise have $\frac{8 \times \overline{d^2-x^2} + 4n^2 \times \overline{d^2-x^2} - \frac{4}{10} \times \overline{2-2n}|^2 \times \overline{d^2-x^2}}{\overline{n+1}|^2} \times \frac{2px}{3}$ $= a$, for *that* of a *Parabolic Spindle*).—Now, for Example's Sake, let a be expounded by $110\frac{1}{2}$ Wine Gallons, or 25525.5 cubic Inches, the corresponding Diagonal *(d)* by 34.4 *nearly*, and n by .85: Then the above general Equation, for a *Spheroidical Cask*, becomes $\frac{10.89 \times \overline{d^2-x^2}}{3.4225} \times .5236x =$ 25525.5, or $1183.36x - x^3 = 15376.8$; whence $x = 17.5$, *nearly*; $\therefore$ $\frac{2\sqrt{d^2-x^2}}{n+1} = 32 =$ the Bung-Diameter, and $\frac{2n\sqrt{d^2-x^2}}{n+1} = 27.2$ $=$ the Head-Diameter; *the very same as in* *Pa.* 208.

COROLLARY 1.

Hence it appears, that, when n vanishes, the above Equation, for a Spheroidical Cask, becomes $\overline{d^2-x^2} \times \frac{16px}{3} = a$, the Equation for a whole Spheroid: Therefore, if a (for Example) be $= 126$ Wine Gallons ($=$ 29106 cubic Inches), and $d = 35.9$ Inches, the corresponding Diagonal, *nearly*, we shall have $x = 5.52$, or 32.82 *(nearly)* for the Semi-lengths of a prolate and an oblate Spheroid respectively; whence 35.47 and 14.55 (*i. e.* $\sqrt{d^2-x^2}$) are respectively the two Semi-diameters thereof; consequently if the Ratio of the Axes of a prolate Spheroid be as 5.52 to 35.47, its Content will be *truly* obtained by the Diagonal Line; but, to effect the same in an oblong Spheroid, the said Ratio must be as 32.82 to 14.55, *nearly*.

COROLLARY 2.

It also appears, that, in the above Equation for a Spheroidical Cask (as well as in *that* for a whole Spheroid), x has two positive Roots; and therefore the other Value of x (in the Equation $1179.9225x - x^3 = 15376.8$) will come out 22.05, *nearly*; and consequently the Bung ($\frac{2\sqrt{d^2-x^2}}{n+1}$) and Head-Diameters ($\frac{2n\sqrt{d^2-x^2}}{n+1}$) are $=$ 28.5 and 24.2 respectively.

N. B.

of that Diagonal, is to the Content of the Cask in Ale or Wine Gallons; so is the Cube of any other Number (or Diagonal) proposed, to the Content of the Cask in Ale or Wine Gallons, answering to that proposed Diagonal: Whence it is evident, that the Cube of the Diagonal of *any* Cask and its Content, (according to this Construction) are always in a constant Proportion; and therefore, if the Content of the Cask first found (or any other), in Ale and Wine Gallons, be divided by the Cube of its Diagonal, we shall obtain (*nearly* .002228 and .00272) two general Multipliers, whereby the Cube of any proposed Diagonal being multiplied, the Product will give (*nearly*) the Content of the Cask (on the Rod) in Ale and Wine Gallons respectively.

The Application of the Diagonal Rod is so very frequent and easy in Cask-Gauging, that it will doubtless be of great Utility to the practical Gauger to be able to know, even by two very easy Operations in Division, when it may be applied, with the utmost Certainty, to any Cask (either of the 1st or 3d Variety) that can ever occur in Practice: For which Purpose, I have taken no small Pains to compute, from the Principles given in the preceding *Note*, the following Table; which exhibits 61 *different* Forms of *Spheroidical Casks*, and 47 *different* Forms of Casks of the *3d Variety*, whose Contents will be truly ob-

E e 2 tained

N. B. All the preceding Investigations, relative to the *Diagonal Line*, are on a Supposition, that every close Cask, now in Use (as well as that from whence it was originally constructed), has strictly this well-known Property; *i. e.* that the Square Root of the Sum of the Squares of half the Sum of the Head and Bung-Diameters and the Semi-length of a Cask is *exactly* equal to the *actual Measure* of its Diagonal; but the latter, when the Rod (or Rule) is bevelled at the End, will (except in some particular Circumstances) exceed the former a small Matter: This small Excess arises from the Heads being a little bevelled all round within, in Order to go into the Groove of the Staves; for, if the Heads were not bevelled within, the above-mentioned Property would be *strictly true* in every close Cask.

tained by the common Diagonal Line: And, from a few general Remarks, delivered in the following Pages, it may be eaſily known whether this Inſtrument exhibits more or leſs than the true Content of a Caſk; provided the *Quotient* of the Head, divided by the Bung-Diameter thereof, is equal to any of *thoſe* in the Table: Which Table, in the Method wherein it is given, might have been extended, from the very ſame Principles, ſo as to have ſhewn treble the Number of *Forms* of Caſks whoſe Contents would be truly obtained by the Diagonal Line; but, as *thoſe* would be ſuch as could never occur in the Practice of Gauging, it would certainly be Time miſapplied to compute them.

I am ſenſible that this will appear, at firſt View, very ſtrange Doctrine to thoſe who have hitherto imagined, *that the Diagonal Line was originally conſtructed from* one *particular Form of Caſks*; nevertheleſs I flatter myſelf, that the *inquiſitive* and *candid* Reader will ſoon be convinced of the *Abſurdity* of ſuch an Opinion. And though I am here under a neceſſity of appearing quite ſingular, as not being ſupported by the Authority of any Author whatever, yet I hope I ſhall not be raſhly condemned *merely* on that Account: For, I dare venture to pronounce, what I have advanced on this Head, however ſtrange and ſingular it may at *firſt* appear, will be found, on the ſtricteſt Examination, to be ſupported by *Truth*; which is the greateſt Authority, and the utmoſt we can aim at in all Mathematical (and other) Enquiries.

I ſhall now proceed to lay down the abovementioned Table, and to give a few explanatory Remarks thereon, in Order to ſhew its Comprehenſiveneſs and Utility in the Application of the Inſtrument.

A TABLE

A TABLE *shewing the Forms of* Spheroidical Casks, *and also those of the* 3d Variety, *whose Contents may be* truly *obtained by the common* Diagonal Line.

The Head-Diameter divided by the Bung-Diameter.	SPHEROIDICAL CASKS.			THIRD VARIETY.		
	The Head-Diameter divided by the Length.			*The Head-Diameter divided by the Length.*		
Quotient.	Quotient.		Quotient.	Quotient.		Quotient.
.60	.328	or	.911	.380	or	.759
.61	.334	or	.910	.387	or	.758
.62	.341	or	.908	.393	or	.756
.63	.349	or	.906	.400	or	.754
.64	.356	or	.903	.407	or	.752
.65	.363	or	.900	.416	or	.750
.66	.370	or	.895	.425	or	.748
.67	.378	or	.890	.434	or	.746
.68	.386	or	.884	.444	or	.744
.69	.394	or	.878	.453	or	.742
.70	.402	or	.872	.462	or	.740
.71	.410	or	.866	.471	or	.738
.72	.418	or	.861	.480	or	.736
.73	.427	or	.856	.490	or	.734
.74	.436	or	.850	.500	or	.732
.75	.444	or	.844	.510	or	.729
.76	.452	or	.838	.520	or	.725
.77	.461	or	.832	.532	or	.720
.78	.470	or	.826	.543	or	.714
.79	.480	or	.820	.556	or	.706
.80	.490	or	.814	.570	or	.690
.81	.500	or	.807	.588	or	.674
.82	.512	or	.800	.612	or	.657
.83	.525	or	.793		.640	
.84	.537	or	.786			
.85	.550	or	.780			
.86	.564	or	.768			
.87	.583	or	.750			
.88	.607	or	.726			
.89	.636	or	.698			
.90 ..	*nearly.*		.670, *nearly.*			

Note. *The preceding Table is now very commodiously graduated on the* Five-Foot Rule, *&c. as made by that accurate Workman, Mr.* Edward Roberts, *in* Dove-Court, Old Jewry.

If

If the Form, or Figure, of any Spheroidical Caſk be ſuch, that the Quotient of the Head-Diameter divided by the Bung-Diameter is .85, and the Quotient of the Head-Diameter divided by the Length is either .55 or .78; then will the *true* Content of ſuch Caſk be obtained by the Diagonal Line: Or, which amounts to the ſame Thing, when the Head-Diameter, Bung-Diameter, and Length are *(very nearly)* equal to, or in the Proportion of, 27.2, 32, and 49.5; or 27.2, 32, and 35 reſpectively.

But if the Quotient of the Head divided by the Bung-Diameter, of *any* Spheroidical Caſk whatever, is .85 (as above), and the Quotient of the Head-Diameter, divided by the Length, ſhould be either leſs than .55, or greater than .78; then will the Diagonal Line exhibit *more* than the true Content of the Caſk: On the contrary, if the ſaid Quotient is between .55 and and .78, the Diagonal Line will *then* give *leſs* than the true Content.

Under the ſecond of the above-mentioned Forms (or *nearly)*, are comprehended all *Rum Puncheons*, *Herefordſhire*, &c. *Cyder Hogſheads*, and half *Hogſheads*; and many other Caſks to be met with in Practice.

If the Form of *any* Caſk of the 3d Variety is ſuch, that the Quotient of the Head-Diameter divided by the Bung-Diameter is .75, and the Quotient of the Head-Diameter divided by the Length is either .51 or .729; then will the Diagonal Line exhibit the *true* Content of ſuch Caſk: Or, which is the ſame Thing in other Words, if the Head-Diameter, Bung-Diameter, and Length are reſpectively equal to, or in the Proportion of, 24, 32, and 47.06, or 24, 32, and 32.92.

But

But when the Quotient of the Head divided by the Bung-Diameter, of any Caſk of the 3d Variety, is .75, and the Quotient of the Head-Diameter divided by the Length is either leſs than .51, or greater than .729; the Diagonal Line will *then* exhibit *more* than the Content of the Caſk; but, when the ſaid Quotient is between .51 and .729, it will ſhew *leſs* than the true Content.

Hence it appears, that the true Contents of *Liſbon* Wine Pipes (being of the 3d Variety) will be nearly obtained by the Diagonal Line, and alſo that it will exhibit more than the Content of *Port Pipes* (of the 3d Variety); becauſe the Quotient of the Head-Diameter divided by the Length is always *leſs* than .51.

It moreover appears, that a *Mountain Butt*, if it is of a ſpheroidical Form, will be ſomewhat under-gauged by the Diagonal Line; but if *it* is of the 3d Variety, the Diagonal Line will *then* exhibit the *true* Content: For the Content of a Caſk of the 3d Variety, whoſe Head, Bung-Diameter, and Length are reſpectively as 26.56, 32, and 41.5 (which is nearly the Form of *Mountain Butts)* will be truly obtained by the Diagonal Line.

To determine whether the Diagonal Line, according to the preceding Table, will exhibit the Content of a Spheroidical Caſk; whoſe Bung-Diameter is 30, Head-Diameter 24.3, and the Length 48.6 Inches.

B. Diam. H. Diam.
30)24.30(.81 *Quotient.*

Length. H. Diam.
48.6)24.30(.5 *Quotient.*

Hence

Hence it appears, becauſe theſe two Quotients *exactly* correſpond in the foregoing Table, that the Content of the Caſk, here propoſed, will be obtained by the Diagonal Rod. —The Truth whereof is thus made out.

The Square of 24.3, half the given Length, is	590.49
The Square of 27.15, half the Sum of the Bung and H. Diameters, is	737.1225
The Sum (or the Square of the Diagonal) is	1327.6125

whoſe Square Root is 36.43 Inches, the required Diagonal of the Caſk; anſwering, as near as can be determined by Inſpection on the Diagonal Line, to $107\frac{3}{4}$ Ale Gallons, or $131\frac{1}{2}$ Wine Gallons. — Now the Content of this Caſk, found by the *general Rule*, *Pa.* 169, is 107.8 Ale Gallons, or 131.6 Wine Gallons; *the ſame as appears by the Diagonal Line.*

If the Quotient of the Head divided by the Bung-Diameter, of any Spheroidical Caſk whatever, be .81 (ſee the preceding *Table)*, and the Quotient of the Head-Diameter divided by the Length be .807; the Diagonal Line will *then* likewiſe ſhew the Content of ſuch Caſk.

But when a Caſk is of the 3d Variety, and the Quotient of the Head divided by the Bung-Diameter is .81 (as above); then, in Order that the Diagonal Line may exhibit its true Content, the Quotient of the Head-Diameter divided by the Length muſt be either .588, or .674: See the *Table.*

It may be neceſſary to obſerve when the Form of a Caſk (of either Variety) is ſuch, that the *Quotient* of the Head-Diameter divided by the Length falls between *thoſe* which correſpond to the

the Quotient of the Head-Diameter divided by the Bung-Diameter (as found in the *Table)*, the Diagonal Line will *then* ſhew *leſs* than the Content of ſuch Caſk: But when the Quotient of the Head-Diameter and Length is either *leſs* than the *leaſt*, or *greater* than the *greateſt* of the aforeſaid Quotients; then will the Diagonal Rod exhibit *more* than the Content of the Caſk. — As for Inſtance, in the Numbers laſt ſpecified, if the Quotient of the Head-Diameter divided by the Length falls between .588 and. 674, the *Diagonal Line* will ſhew *leſs*; but, if the ſaid Quotient is leſs than .588, or greater than .674, *it* will exhibit *more* than the Content of ſuch Caſk.—Obſerve the ſame of all the other Numbers in the preceding *Table.*

I might here have given, from the very ſame general Principles, a *Table* of the 2d *Variety* of Caſks, or thoſe of a *Parabolic Spindle:* But ſuch a *Table*, I judged, would be wholly unneceſſary; ſince it is well known, that the Content of a Caſk of this Variety differs but little from that of a ſpheroidical Form, each having the ſame Head-Diameter, Bung-Diameter, and Length; eſpecially ſuch Caſks where the Diagonal Rod can be applied; and therefore a *Table* for the 2d *Variety* would differ but very little from that already given for *Spheroidical Caſks.*

Before I put an End to this *Section*, it may be proper to remark, that the Form of a Caſk, with Reſpect to Curvature, depends chiefly upon the Diſtance *mn* (ſee *Fig. Pa.* 168); which Diſtance (it is well known to every *Cooper)* may be made greater or leſs to any aſſigned Head-Diameter, Bung-Diameter, and Length: For the Sides of the Staves of every cloſe Caſk are jointed upon an Inſtrument (called a *Jointer)*, which is not a

perfect *Plane*, but is, in a ſmall Meaſure, concave (or hollow) each Way from the *Jointer-Iron*; by which Means the two Sides of every Staff, from each Extremity thereof to the Middle, have a ſmall Degree of Curvature; the Vertex of the Curve being at (or near) $\frac{1}{4}$th of the Staff's Length: And therefore, according as the Sides of the Staves have a greater or leſs Degree of Curvature, or the *Jointer*, from whence they were formed, is more or leſs hollowed, the Diſtance *mn* (ſee *Fig. Pa.* 168.) will be leſs or greater, or (as the *Coopers* phraſe it) the Caſk will have a *higher* or *lower* Quarter.

By Reaſon of theſe plain Facts, I therefore cannot coincide with the Opinions of thoſe ingenious Gentlemen who have aſſerted, that ſcarce any cloſe Caſk can be made to contain ſo much as the *Middle Fruſtum* of a *Spheroid*. — It muſt, indeed, be allowed, that there are more Caſks to be met with that are *leſs*, than are *equal* to that Variety: But it does not, however, follow from thence, that it is *impoſſible* to form a cloſe Caſk equal to the *Middle Fruſtum* of a *Spheroid*, or even much *greater* than that Variety, if it was required: And therefore it is highly requiſite, that every Perſon, concerned in Caſk-Gauging, ſhould have a ſtrict Regard to the Quarter of the Caſk, or the Diameter in the Middle between thoſe of the Bung and Head; which Diameter may be, *very nearly*, determined, by finding the Diſtance *mn*. (See *Pa.* 168).

SECTION

SECTION XI.

Of Ullaging of Casks.

THE Method now practiſed in the Ullaging of Caſks, whether lying or ſtanding, is by the Lines of Segments on the Sliding-Rule (deſcribed in *Pa.* 32, *&c.*) Though other Methods may indeed be given, far more general and accurate; yet there are none, that have occurred to me, but what will, perhaps, be thought too tedious for practical Uſe.

The late ingenious Mr. *William Yeo* (in a whole Treatiſe upon Ullaging, publiſhed in the Year 1749) has computed very accurate, extenſive, and familiar Tables of Segments, not only for one particular Sort, but for eight different Forms, both of ſtanding and lying Spheroidical Caſks: From theſe *Tables* we can readily determine what two Forms of Spheroidical Caſks agree, *very nearly*, with the Lines of Segments on the Sliding-Rule. — *Viz.* The Ullage of every ſtanding Spheroidical Caſk, whereof the Quotient of the Head divided by the Bung-Diameter is .82,* will be,

* It appears, in the Tables above cited, that the 6th Column for ſtanding Spheroidical Caſks (where the Head divided by the Bung-Diameter is .81, .82, .83, or .84) anſwers beſt to the Line S.S on the Sliding-Rule; therefore let .82 (which is near the Mean of the Four) be taken for the Quotient of the Head divided by the Bung-Diameter; then will the Ullage of *any* ſtanding *Spheroidical Caſk*, having that Property, be (*very nearly*) obtained by the Sliding-Rule, *let the Length of the Caſk be what it will*: For we ſhall prove in a *Corollary* farther on (*Pa.* 225), that in *any* two ſtanding *Spheroidical Caſks*, having the ſame Ratio of Bung and Head-Diameters, and alſo the Quotient of the wet Inches of each Caſk, divided by

very nearly, obtained by the Line S.S, and thoſe marked A and B on the Sliding-Rule: And the Ullage of a lying Spheroidical Caſk will be *nearly* found by the Sliding-Rule, when the Quotient of the Head divided by the Bung-Diameter is 75, and the Quotient of the Head-Diameter divided by the Length is .5†. Moreover, it is eaſy to perceive, from the fore-mentioned Tables, that, in any ſtanding Spheroidical Caſk, if the Quotient of the Head divided by the Bung-Diameter is leſs than .82, the Ullage then obtained by the Lines S.S and A and B, will be *too much*, if the Caſk is leſs than half full; and *too little* if above half full: But, on the contrary, if the ſaid Quotient is greater than .82; then the Ullage, found as above, will be *too little*, if the Caſk is leſs than half full; and *too much*, if more than half full.

Again,

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by its reſpective Length, equal to each other, the Ullages of thoſe two Caſks will be in the Ratio of their whole Contents.

But, with Regard to lying Caſks, the Caſe is different: For the Ullages of any two lying *Spheroidical Caſks*, having different Lengths, and the Bung and Head-Diameters, and alſo the wet Inches, the ſame, will not (except when the Caſks are *exactly* half full) be to each other in the Ratio of the whole Contents of thoſe Caſks. Hence it appears, that the Line S.L on the Sliding-Rule can only anſwer to *one* particular Form of *Spheroidical Caſks*, *i. e.* ſuch, whoſe Head, Bung, and Length, are in ſome conſtant Ratio: In Order to determine which, proceed as follows.

† In the aforeſaid Tables, for lying *Spheroidical Caſks*, it appears that the Segments which anſwer neareſt to thoſe on the Sliding-Rule, ſtand under theſe Ratios of the Bung and Head-Diameters; *viz.* .74, .75, and .76: Suppoſe, for Inſtance, we take .75 (as being the Mean); then, by the

well-known *Theorem* for *Spheroidical Caſks*, $\overline{2 \times .75^2} \times .7854 \times \frac{l}{3} = 1$;

whence .6708 $l = 1$, and $l = \frac{1}{.67} = 1.5$, the required Length, *nearly*.

Hence, the Quotient of the Head divided by the Bung-Diameter is .75, and the Quotient of the Head-Diameter divided by the Length $\left(\frac{.75}{1.5}\right)$ is .5: Or, which is the ſame Thing, the Head, Bung-Diameter, and Length, are as 24, 32, and 48; whence the Ullage of every lying *Spheroidical Caſk*, having this Proportion, will be *nearly* obtained by the Sliding-Rule.

Again, it appears by these Tables, that in a lying Spheroidical Cask, if the Quotient of the Head divided by the Bung-Diameter be less than .75, and the Quotient of the Head-Diameter divided by the Length be less than .5, the Ullage found by the Line S.L and the Lines A and B, will be *too much*, if the Cask is less than half full; and *too little*, if above half full: But if the said Quotients are greater than .75 and .5; then the Ullage, found as above, will be *too little*, if the Cask is less than half full; and *too much*, if above half full.

It may be proper to observe to the practical Reader, that, in ullaging by the Sliding-Rule, we are much more subject to Error in lying, than in standing Casks.

To find the Ullage of a Cask by the SLIDING-RULE.

PROP. I.

The Length (or Bung-Diameter), wet Inches, and Content, of a standing (or lying) Cask being given; to find the Ullage thereof.

GENERAL RULE.

To 100 on S.S (or S.L) set the Length (or Bung-Diameter) on the Slide N; then against the wet Inches on N, is the *Segment* on S.S, if a standing Cask; or on S.L, if a lying Cask: Again, to 100 on A, set the whole Content on B; then opposite the said Segment on A, is the required Ullage on B.

EXAMPLE

EXAMPLE 1.

Let the Content of a ſtanding Caſk be 142 Gallons, the Length 52, and the wet Inches 20; to find the Ullage, or Quantity of Liquor in the Caſk.

To 100 on S.S, ſet the Length 52 on N; then againſt the wet Inches 20 on N, is 37 the Segment on S.S: Again, to 100 on A, ſet the whole Content 142 on B; then againſt the above Segment 37 on A, is $52\frac{1}{4}$ Gallons, the required Ullage on B.

EXAMPLE 2.

Suppoſe the Content of a lying Caſk to be 112 Gallons, the Bung-Diameter 33, and the wet Inches 22.5; to find the Ullage.

To 100 on S.L, ſet the Bung-Diameter 33 on N; then againſt the wet Inches 22.5 on N, is 74.5 the Segment on S.L: Again, to 100 on A, ſet the Content of the Caſk 112 on B; then againſt the ſaid Segment 74.5 on A, is 83.5 Gallons, *very nearly*, on B.

After the Segment of a Caſk (either ſtanding or lying) is found, the Reſult, from the Remainder of the Operation, will be the very ſame as above; if to 100 on A, we ſet the Segment (inſtead of the Content of the Caſk) on B, and look againſt the ſaid Content on A, for the Ullage on B.*

In

* The Ullages of two ſtanding Spheroidical Caſks (whoſe Bung and Head-Diameters are to each other in the ſame Ratio, and the wet Inches of each Caſk in the Ratio of their Lengths) will be to each other as their whole Contents (ſee Pa

In the last Example, the Segment was found to be 74.5: Therefore to 100 on A, set 74.5 on B; then against the Content 112 on A, is 83.5, *very nearly*, on B; *the same as before.*

It may not be amiss to observe, that after the Segment of any Cask is found (as above) by the Lines S.S and S.L, the Rest of the Operation may be performed, without the Lines A and B, by only multiplying the whole Content of the Cask by the said Segment; the Product thereof, after pointing off two Decimals more than are contained in both the Content and Segment, will be the required Ullage.

In the preceding Example, the Content of the Cask is 112 Gallons, which being multiplied by the Segment 74.5, the Product is 8344.0; therefore, by cutting off two Decimals more, we have 83.440 Gallons, the required Ullage; *the same as above.*

If it was required to find the Vacuity, or the Quantity of Liquor drawn out of a Cask, the Method of Operation will be the very same as any of those given above; only observe to use the dry Inches, instead of the Wet.

PROP.

Pa. 225): Therefore let a denote the Content of a Cask, whose Ullage is sought, and b the Segment (or corresponding Ullage) of a Cask, whose Content is 100 Gallons; we shall then have, $100 : a :: b : \frac{a \times b}{100}$, or, alternately, $100 : b :: a : \frac{a \times b}{100}$: Whence it is plain, that the Result will be the very same, whether a or b (on B) is set to 100 on A: Moreover, dividing the Product $(a \times b)$ by 100, is manifestly the same as cutting off two Decimals more than are contained in the Factors a and b.

The same is to be observed in two lying Casks, only those indeed must be similar in every Respect, and consequently the Segments (or Ullages) will be so too; provided the wet Inches and Bung Diameter of each Cask are to each other in the same Ratio.

PROP. II.

The Bung and Head-Diameters, Length, and wet Inches of any standing Spheroidical Cask *being given; to determine the Ullage thereof.*

RULE.

1. Divide the Square of the wet Inches by three times the Square of half the Length of the Cask, to the Quotient add Unity, and from this Sum subtract the Quotient of the wet Inches divided by half the Length of the Cask, and *note* the Difference.

2. Multiply the Sum of the two given Diameters by their Difference, that Product multiply by the wet Inches, and this Product multiply by the above *noted* Difference; then let this last Product be subtracted from the Square of the Bung-Diameter multiplied by the wet Inches,† and the Remainder being multiplied by $\left\{\begin{array}{l}\text{.0027851 for Ale Gallons}\\ \text{.0034 for Wine Gallons}\end{array}\right\}$, or divided by $\left\{\begin{array}{l}\text{359 for Ale Gallons}\\ \text{294 for Wine Gallons}\end{array}\right\}$ the Product, or Quotient, gives the required Ullage.

EXAMPLE.

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† Let the Bung-Diameter (CD, *Fig.* XIV.), Head-Diameter (EF or GH), and half the Length (OL or OM) of a *Spheroidical Cask* (in *Inches*) be denoted by b, h, and l respectively; also let $p = .78539$, the Semi-transverse Axis (AO) of the whole Spheroid $= v$, and the variable Distance $Lm = x$: Then, by the Property of the Curve, we have $\frac{pb^2}{v^2} \times \overline{v-l+x} \times \overline{v+l-x}$ $(= \frac{pb^2}{v^2} \times \overline{v^2-l^2+2lx-x^2}) =$ the Measure of the Section nr; therefore $\frac{pb^2}{v^2} \times \overline{v^2\dot{x}-l^2\dot{x}+2lx\dot{x}-x^2\dot{x}}$ is the Fluxion of the

EXAMPLE.

Let the Bung-Diameter of a Spheroidical Cask be 35 Inches, the Head Diameter 28.7, the Length 40, and the wet Inches 30; required the Ullage in Ale and Wine Gallons.

 OPERATION.

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the required Solid, whose Fluent is $\frac{pb^2}{v^2} \times \overline{v^2x - l^2x + lx^2 - \frac{x^3}{3}}$; but, by the Nature of the Curve, $b^2 = \frac{b^2}{v^2} \times \overline{v^2 - l^2}$, $\therefore v^2 = \frac{b^2l^2}{b^2 - b^2}$; which being substituted (for v^2) in the above Expression, we get $\frac{pb^2}{\frac{b^2l^2}{b^2-b^2}} \times \overline{\frac{b^2l^2x}{b^2-b^2} - l^2x + lx^2 - \frac{x^3}{3}}$ $\left(= \frac{p \times \overline{b^2 - b^2}}{l^2} \times \overline{\frac{b^2l^2x}{b^2-b^2} - l^2x + lx^2 - \frac{x^3}{3}} = pb^2x + \frac{p \times \overline{b^2 - b^2}}{l^2} \times \overline{lx^2 - l^2x - \frac{x^3}{3}}\right) = pb^2x + px \times \overline{b^2 - b^2} \times \overline{\frac{x}{l} - 1 - \frac{x^2}{3l^2}}$, the Content (in Inches) of the variable Part FEnr.

Q. E. I.

COROLLARY 1.

When, in the foregoing Expression for the Ullage, $x = 2l$; we then get $2pb^2l + 2pl \times \frac{b^2 - b^2}{3}$, or its Equal, $\frac{2pl}{3} \times \overline{2b^2 + b^2}$, for the whole Content of a Spheroidical Cask, in *cubic Inches*.

COROLLARY 2.

Hence we can easily deduce the Reason of that Assertion in the *Note*, Pa. 219.—For it is evident (supposing $\frac{x}{l}$ to remain the same) that $\frac{x}{l} - 1 - \frac{x^2}{3l^2}$ will be a constant Quantity; and therefore (b and b being constant) the Expression ($pb^2x + px \times \overline{b^2 - b^2} \times \overline{\frac{x}{l} - 1 - \frac{x^2}{3l^2}}$) for the Ullage, let l be what it will, is evidently as x, the Altitude of the Segment; but x and l (by Hypothesis) are in a constant Ratio, and therefore the above Expression (in such

OPERATION.

The Square of 30 (the wet Inches) is 900, which being divided by 3 times the Square of 20, half the Length of the Cask (*viz.* 1200), the Quotient will be .75, to which add Unity, and it becomes 1.75
Subt. the Quotient of 30 divided by 20, *viz.* 1.5

Difference *note* .25

The

ſuch Caſe) will be as l, the whole Length of the Caſk, which is manifeſtly as the whole Content thereof, b and b remaining the ſame; conſequently the above Expreſſion (for the Ullage of any upright *Spheroidical Caſk*) when b, b and $\frac{x}{l}$ remain the ſame, will be to the whole Content thereof in a conſtant Ratio, let its Length be what it will.

OTHERWISE, *more generally*.

Let b and b denote any two Numbers whatever, in a conſtant Ratio to each other, and let $\frac{x}{l}$ be ſuppoſed a *conſtant* Quantity. Put $\frac{x}{l} - 1 - \frac{x^2}{3l^2}$ (being in this Caſe conſtant) $= n$; then the foregoing Expreſſion for the Ullage becomes $px \times \overline{b^2 + \overline{\overline{b^2 - b^2}} \times n}$; but the whole Content of the Caſk is (by *Cor.* 1.) expreſſed by $\frac{2pl}{3} \times \overline{2b^2 + b^2}$; we muſt therefore prove whether or not $px \times \overline{b^2 + \overline{b^2 - b^2} \times n}$ is to $\frac{2pl}{3} \times \overline{2b^2 + b^2}$ in a conſtant Ratio, under the above-mentioned Circumſtances. Firſt then (by Hypotheſis) $px : \frac{2pl}{3}$ (or $x : \frac{2l}{3}$) in a conſtant Ratio; it therefore now remains to prove, that $b^2 + \overline{b^2 - b^2} \times n$ (or $b \times b + \overline{b+b} \times \overline{b-b} \times n$) is to $2b \times b + b \times b$ in a conſtant Ratio: Now it is evident, that every Factor (*i. e.* b, $b+b$, $b-b$, and b) contained in the Terms of the Ratio will be equally affected by any Multiple, or Part, of b (and b) being taken; whence it follows, that the four Quantities, or Rectangles ($b \times b$, $\overline{\overline{b+b} \times \overline{b-b}} \times n$, $2b \times b$, and $b \times b$), *viz.* two in each Term of the propoſed Ratio, will be each

of

The Sum of the two Diameters is	63.7
Multiplied by their Difference	63.3
	1911
	3822
Gives	401.31
Multiplied by the wet Inches	30
Gives	12039.30
Multiplied by the above *noted* Diff.	.25
The Product is	3009.825

The Square of the Bung-Diam. (35) is	1225
Multiplied by the wet Inches	30
Product is	36750
From which take the last Product	3009.825
Remains	33740.175,

which being multiplied (or divided) according to the preceding Rule, we shall have 94 Ale Gallons

and

of them augmented, or diminished, alike; and consequently $b \times b + \overline{b+b} \times \overline{b-b} \times n : 2b \times b + b \times b$ in a constant Ratio; this being multiplied by two Factors, which are (by Hypothesis) also in a constant Ratio to each other (*i. e.* px and $\frac{2pl}{3}$), the Products must also be in a constant Ratio; that is, $px \times \overline{b^2 + \overline{\overline{b^2 - b^2}} \times n} : \frac{2pl}{3} \times \overline{2b^2 + b^2}$ in a constant Ratio, let b and b be what they will (in a constant Ratio), and $\frac{x}{l}$ a constant Quantity. —That is, in Words, let any two standing Spheroidical Casks be taken, whose Bung-Diameters and Head-Diameters are in the same Ratio to each other, and also let the wet Inches of each Cask be to each other in the Ratio of their Lengths; then will the Contents of those Ullages be to each other in the Ratio of the whole Contents of the Casks, let their Lengths be what they will.

and 114.7 Wine Gallons, the Contents of the required Ullage.

The whole Content of the foregoing Cask (by the *Rule* in *Pa.* 169) is 148.5 Wine, and 121.5 Ale Gallons; whence, by the *Sliding-Rule*, the Contents of the Ullage will come out the very ſame as thoſe above: The Reaſon whereof is, becauſe if the Head-Diameter (28.7) is divided by the Bung-Diameter (35), the Quotient will be .82. (See *Pa.* 219).

The preceding Rule is *ſtrictly true* for determining the Ullage of any ſtanding Spheroidical Cask whatever; and, though rather too tedious for ordinary Practice, will, I apprehend, be found more expeditious than any *general Rule* hitherto given for that Purpoſe; there being no Neceſſity, by this Method, for previouſly finding the Content of the Cask, before *that* of the Ullage.

But, if there be known (beſides the Dimenſions given in the foregoing Propoſition) the Diameter of the Liquor's Surface, we can readily determine the Ullage of any upright Cask, let its Variety be what it will.

For let a Mean-Diameter, and conſequently the Area in Ale and Wine Gallons, correſponding to the Bung-Diameter and the Diameter of the Liquor's Surface, be found, agreeable to the Variety of the Cask, as already taught in *Sect.* X; then this Area being multiplied by the Diſtance of the Surface of the Liquor from the Bung-Diameter, and that Product added to, or ſubtracted from half the Content of the Cask, according as it is *more* or *leſs* than half full; the Sum, or Difference, will be the required Ullage.

From what has been delivered (*Sect.* IX. *Pa.* 179), we might eaſily deduce Rules for computing the Diameter of the Surface of the Liquor, at any given

given Altitude of an upright *Spheroidical Cask*, or that of a *Parabolic Spindle:* But the following Method is far more expeditious, and will be sufficiently exact, for any of the three Varieties.

Suppose, for Example, the Distance *br* (see *Fig. Pa.* 165) to represent the wet Inches; then carefully measure the perpendicular Distance *ad*, the Double whereof being taken from the Bung-Diameter AB, leaves the Diameter of the Liquor's Surface.

Let, for Instance, the Bung and Head-Diameters, Length and wet Inches of a *Spheroidical Cask* be the same as in the preceding Example, also let the Diameter of the Liquor's Surface be 33.5 Inches, found (in this Case) by the Rule in *Pa.* 179; required the Ullage in Wine Gallons.

Operation.

Bung. Head. Quot.
35) 33.50 (.96, *nearly*.

Then (by the *general Rule*, *Pa.* 196) against .96, in the Column for Spheroidical Casks, we have 67

Which being multiplied by the Differ- of the Diameters . . . . . . } 1.5

335
67

Gives 1.005

Add the Head-Diameter 33.5

Mean-Diameter 34.505, the Area

Area whereof in Wine Gallons is 4.04, &c.

Multiplied by the Distance of the Liquor's Surface from the B. Diam. } 10

Gives 40.40

Add half the Content (see Pa. 228) 74.25

Gives the Ullage 114.65 Wine Gallons; *the same as before.*

The Business of finding a Mean-Diameter (*Sect.* X.) being now rendered very exact, expeditious, and general, for any of the three Varieties; it is therefore presumed, that the preceding *Method* of computing, by the Pen, the Ullage of any standing Cask (and also that which follows for lying Casks), will be found preferable to any *other* that can be given.

PROP. III.

The Bung and Head-Diameters, Length, Variety, and wet Inches of any lying Cask (less than half full) being known; to find the Quantity of Liquor therein, in Ale and Wine Gallons.

RULE.

Let the Mean-Diameter be found (see *Sect.* X.) agreeable to the proposed Variety of the Cask: From the wet Inches subtract half the Difference between the Bung and Mean-Diameter, and divide the Remainder (with Cyphers annexed, see the Rule in *Pa.* 93) by the Mean-Diameter; then, against the Quotient, under the Letter V.S, in the Table of the Areas of the Segments of a Circle, we have a Decimal Fraction, which being multiplied by the Square of the Mean-Diameter, that

that Product multiply by the Length of the Cask, and this last Product divide by 282 for Ale, and 231 for Wine Gallons, the Quotient will give the Ullage sought.*

EXAMPLE.

Suppose the Bung-Diameter of a *Spheroidical Cask* is 32 Inches, the Head-Diameter 24, the Length 48, and the wet Inches 14; required the Ullage in Ale and Wine Gallons.

OPERATION.

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* Let BCEF (*Fig.* XV.) represent a lying Cask, *ab* its Mean-Diameter, A*d* the wet Inches: Then, supposing RA drawn parallel to the Axis *sn*, it is plain that $Ae\ (=bc) = \frac{AD - ab}{2} = \frac{1}{2}$ the Difference between the Bung and Mean-Diameter; therefore $Ad\ (cr) - Ae = br =$ the Versed Sine of the Segment of a Circle whose Diameter is *ab*. Now let the Measure of a Segment of a Circle, whose Diameter is Unity, (in the present Case = 1 Inch) be denoted by A, the Mean-Diameter $ab = b$, and the Length $sn\ (= vw) = l$: Then, by the *Theorem*, Pa. 91, $1^2 : b^2 :: A : b^2 \times A =$ the Measure of a Segment of a Circle (whose Diameter is *ab*) similar to that of A; consequently $b^2 A \times l =$ the Measure (in Inches) of the Ullage AB*vw*F. Q. E. I.

COROLLARY.

If A represents the Measure of a Segment of a Circle, whose *Area* is Unity (*i. e.* one Inch), *b* and *l* as before: Then, because the Areas of Circles are as the Squares of their Diameters, we have $1 : b^2 \times .7854 :: A : .7854 b^2 \times A =$ the Measure of a Segment, similar to that of A; therefore $.7854 b^2 l \times A =$ the Measure of the Ullage AB*vw*F in Inches, *nearly*; that is, the Segment in the Table, in *Everard*'s Gauging (where the Area of the Circle is Unity), being multiplied by the whole Content of the Cask, gives the required Ullage: But the Methods exhibited above are more expeditious; because we are, by those Methods, under no Necessity of, previously, finding the whole Content of the Cask; and moreover, the Ullage may be obtained with the same Expedition, whether the Cask is more or less than half full, provided the Table of Segments of a Circle was continued to 1000 Places, or to the Area of the whole Circle; *i. e.* to .785398, &c. Which indeed may be very easily effected, in the following Manner.

From .785398 (the Area of a Circle whose Diameter is Unity) subtract, successively, the Segments answering to the Versed Sines .499, .498, .497, and .496, &c. and the Remainders will shew, respectively, the Measures of the Segments corresponding to .501, .502, .503, and .504, &c. Parts of Unity, or the Diameter of the Circle.

OPERATION.

The Bung-Diameter 32
The M. Diam. by the Rule Pa. 196, is 29.57

Difference 2.43

Half Difference is 1.21,
which being taken from 14, the wet Inches, leaves 12.79; then

M. Diam. M. Wet. Quot.
29.57) 12.79000(.432

Against .432, under the Letters V.S, in the Table of Segments of a Circle, is .324909
Multiplied by the Square of 29.57, the Mean-Diameter; *viz.* . . } 874.38

Product is 284.0939
Multiplied by the Length 48

22727512
11363756

Product 13636.5072

282)13636.507(48.356 Ale Gallons.
231)13636.507(59.032 Wine Gallons.

But if it be required to find the Quantity of Liquor drawn out of any lying Cask, when less than half full, or remaining in it when more than half full, proceed as follows.

Find, by the preceding Rule, the Circular Segment in the *Table* corresponding to the wet Part of the Cask, when less than half full, or to the dry Part, when more than Half; which being subtracted from .785398, the Remainder will be the Measure of a Segment similar to the wet and dry

dry Part of the Cask respectively; this being multiplied and divided, as in the preceding Rule, gives the Ullage sought.

Let it be required to find, in the foregoing Example, the Vacuity, or Quantity of Liquor drawn out of the Cask; the Operation will be as follows.

From the Area of a Circle, whose Diameter is Unity, *viz.*	.785398
Subtract the Segment answering to the wet Part (see *Pa.* 232.)	.324909
Leaves a Segment similar to that corresponding to the dry Part of the Cask	.460489
Multiplied by the Square of the Mean-Diameter; *viz.*	.874.38
Product is	402.6423
Multiplied by the Length	48
	32211384
	16105692
Product	19326.8304

282)19326.830(68.534 Ale Gallons.
231)19326.830(83.665 Wine Gallons.

The Method of Operation for finding the Quantity of Liquor in a lying Cask, more than half full, is the very same as *that* given above; except, that the Segment (in the Table) must be found for the dry, instead of the wet Inches.

The foregoing *Method* of ullaging a lying Cask, though not strictly true, is more exact than any *other* that has yet occurred to me, and may be

 applied,

applied, with equal Facility, to any of the three Varieties; becauſe it chiefly depends on the Mean-Diameter, which is now obtained with great Eaſe and Exactneſs, let the Variety of the Cask be what it will: (See *Sect.* X.)—It may, however, be proper to obſerve here, that the Quantity of Liquor in a lying Cask, obtained by the preceding Methods, will be *too much*, if it be more than half full; and *too little*, if leſs than half full; but the greateſt Error that can poſſibly happen, either in Exceſs or Defect, will be wholly inconſiderable in the Practice of Gauging.*

* It is very evident (ſee *Fig.* XV.) that, when the wet Inches are equal to (or leſs than *Ae*) half the Difference between the Bung and Mean Diameter, the Verſed Sine, and, conſequently, the mean wet Inches vaniſh; and therefore the Quantity of Liquor in the Caſk (according to this Method of finding the Ullage) will be $=0$, when the wet Inches are equal to *Ae*, or leſs than that Diſtance: Which is *abſurd*. — Whence it follows that the Quantity of Liquor in a Caſk (obtained by this Method) will be a *ſmall Matter too little*, if leſs than half full; and *too much*, when above half full.

SECTION

SECTION XII.

OF *meaſuring* CURVE-LINED PLANES, *by Approximation.*

THE general Method of approximating the Areas of curvilineal Planes, by Means of any given Number of equidiſtant perpendicular Ordinates (or Diameters), was firſt demonſtrated by the moſt illuſtrious NEWTON, and is well known to be a Subject of very great Importance in ſpeculative Mathematics.

And although this general Method has already been adapted to the preſent Subject (particularly, firſt of all, by that excellent Mathematician the late Mr. *Robert Shirtcliffe*, in his Theory and Practice of Gauging, and afterwards by my late worthy and ingenious Friend, Mr. *Samuel Farrer*, in the Appendix to *Overley*'s Gauging), yet we find it has not ſufficiently merited the Attention of every practical Gauger, which, it is preſumed, is owing to the Tedioufneſs of the Rules hitherto laid down.

For this Reaſon, I have endeavoured to put the Matter in as *clear* and *practical* a Light, as the Nature of the Subject can poſſibly admit of, and have illuſtrated the ſame with ſuitable Examples: And moreover, to oblige the ſpeculative Readers, I have given, in the ſubjoined Note,* the

PROPOSITION.

* *Suppoſe the* black Curve-line *vnp* (ſee Fig XVI.) *to repreſent a ſmall Portion of any Curve whatever, and the* dotted Line *vnp a ſmall Portion of a* common

Demonſtration of this Method; (which indeed does not eſſentially differ, except in one Circumſtance, from

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common Parabolic Curve, each paſſing through the Extremities of the three equidiſtant Perpendiculars Av, Bn, Cp: *To find an Expreſſion in Terms of thoſe three Ordinates* (or Diameters), *and their common Diſtance* AB (BC, &c.), *that ſhall* accurately *meaſure the Parabolic Space*; *and conſequently that comprehended by the black Curve-line* vnp, *the Right-line* AC, *and the Perpendiculars* Av, Cp, indefinitely near.

Suppoſe the Axis (PQ) of the Parabolic Curve, to be parallel to the Ordinates of the propoſed Curve, draw the Right-line (or Ordinate) vwp, and parallel thereto draw MnS; which is well known to be a Tangent to the Parabolic Curve, at the Point n; produce Av and Cp to meet MS in m and s: Then becauſe it is proved, by the Writers on Fluxions, that a Parabola is two-thirds of a Rectangle of the ſame Baſe and Altitude; it follows, from the very ſame Principles, that the Parabolic Area $vnpwv$, is two-thirds of the Parallelogram $vmsp$. — Now it is evident, from common Geometry, that $Bn\ (=\frac{Am+Cs}{2}) \times 2AB$ is equal to the Area of the quadrilateral Space $AmsC$, and alſo that the Area of the Quadrilateral $AvwpC$ is expreſſed by $\overline{Av+Cp}\ (2Bw) \times AB$; moreover it is plain, that the Quadrilateral $AmsC$ is greater than the Parabolic Area $AvnpC$, by *exactly* half what the Quadrilateral $AvwpC$ wants of that Area; conſequently twice $AmsC$ (= twice the Parabolic Space $AvnpC + vnpw$, or twice $vmnsp$) added to $AvwpC$, gives three times the Parabolic Area $AvnpC$; which Area alone, will therefore be, *accurately*, expreſſed by $\frac{2Bn \times 2AB + \overline{Av+Cp} \times AB}{3}$, or $\overline{Av+4Bn+Cp} \times \frac{AB}{3}$; and therefore, when the three Ordinates (or Perpendiculars) are taken pretty near to each other, the common Parabolic Curve paſſing through their Extremities, will, *very nearly*, coincide with any other Curve, paſſing through the ſame three Points: Becauſe, as a *Parabolic Curve* has an infinite Variation of Curvature, it may be juſtly conceived to be, *very nearly*, coincident with any *other Curve* for a ſmall Diſtance: Whence it is plain, the above Expreſſion ($\overline{Av+4Bn+Cp} \times \frac{AB}{3}$) will exhibit the *accurate Meaſure* of the Parabolic Space $AvnpC$; and conſequently, *very nearly* (or accurately, under ONE particular Circumſtance), of *that* bounded by the Right line AC, the Perpendiculars Av, Cp; and any other Curve-line paſſing through the Extremities (v, n, and p) of the three equidiſtant Perpendiculars Av, Bn, and Cp. Q. E. D.

COROLLARY I.

From hence it is eaſy to deduce a *general Rule* for determining, *very nearly*, with any odd Number of equidiſtant Ordinates or Perpendiculars whatever, the Meaſure of any curvilineal Plane, bounded at its Ends by Right-lines parallel

from what is given by that incomparable and most profound Mathematician, the late Mr. THOMAS SIMPSON, in his *Dissertations*, *Pa.* 109).

And

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parallel to each other: For let the perpendicular Distance of those two given parallel Lines Av, Gb, be divided into equal Parts, by any odd Number of Perpendiculars, Bn, Cp, De, *&c.* which in *some Curves* are considered as Ordinates, and in *others* (particularly the *Parabola*) as Diameters (and the greater the Number taken, the greater will be the Degree of Accuracy, in every curvilineal Plane, except the *Parabola*, wherein it is *strictly* true): Then by the very same Reasoning, as we found $\overline{Av+4Bn+Cp} \times \frac{AB}{3}$, for the Area of $AvnpC$, we get $\overline{Cp+4De+Ef} \times \frac{CD}{3}$ $(\frac{AB}{3})$ for the Area of $CpefE$, and likewise that of $EfgbG = \overline{Ef+4Fg+Gb} \times \frac{AB}{3}$, *&c.* therefore the Sum of those Areas (each having one common Multiplier) will be expressed by $\frac{AB}{3} \times \overline{Av+4Bn+2Cp+4De+2Ef+4Fg+Gb} =$ the Area of the curvilineal Plane $AvnpefgbG$, *nearly*: Whence the general Rule is manifest.

COROLLARY 2.

If, at the Extremity of any curvilineal Space whose Area is sought, the Ordinate is supposed to vanish; then (Av being Nothing) we have $\overline{4Bn+2Cp+4De+2Ef+4Fg+Gb+\&c.} \times \frac{AB}{3}$, a general Expression for approximating, with any even Number of equidistant Perpendiculars, the Area of any curvilineal Plane, bounded by two perpendicular Right-lines and a Curve. The above Expression, in Words, will be as follows.—*To four times the Sum of the 1st, 3d, and 5th,* &c. *Perpendiculars* (beginning at the least) *add the last, and also Double the Sum of all the Rest; this Total being multiplied by $\frac{1}{3}$d of their common Distance, the Product will be the required Area,* nearly.

COROLLARY 3.

Hence it appears, that if, at both the Extremities of any curvilineal Plane, the Ordinates (or Perpendiculars) be supposed Nothing, we shall have $\overline{4Bn+2Cp+4De+2Ef+4Fg+\&c.} \times \frac{AB}{3}$, a general Expression for determining, with any odd Number of Ordinates, the true Measure, *nearly*, of any curvilineal Plane, bounded by one Right-line and a Curve, or wholly by a Curve. The Expression, in Words, will be thus.—*To four times the Sum of the 1st, 3d, 5th,* &c *Perpendiculars, add Double the Sum of all the Rest; this Total being multiplied by $\frac{1}{3}$d of the common Distance of the Perpendiculars, the Product gives the Area sought,* nearly.

SCHOLIUM.

And from the said Demonstration, I have deduced two *general Rules*; *one* for approximating, with any even Number of equidistant perpendicular Ordinates, the Area of any curvilineal Space, bounded by two perpendicular Right-lines and a Curve; and the *other* for obtaining, with any odd Number equidistant perpendicular Ordinates (or Diameters), the *true* Area, *very nearly*, of any curvilineal Space, comprehended either by a Right-line, and a Curve, or wholly by a Curve: But it is to be observed, that these Rules, for the most Part, will not approximate the Areas, of such Planes as occur in the Subject of Gauging, so

SCHOLIUM.

* The two preceding Rules may be applied, with the greatest Facility and Advantage, not only to the Mensuration of Land, but also to that of several Kinds of Artificers Work; such as *Glazing*, *Painting*, *Paving*, *&c.* For if the Boundary of any plane Figure whatever, whose Measure is sought, should consist partly of Right-lines, and partly of Curves whose Properties are unknown; such Figure may then be divided into Triangles and curvilineal Figures, each Figure bounded either by a Right-line and a Curve, or by two perpendicular Right-Lines and a Curve: And therefore, any assigned Number of equidistant Perpendiculars may be found by actual Dimensions, the Measures of those curvilineal Figures may, in many Cases with 5 or 6 equidistant Perpendiculars, be approximated to an amazing Degree of Exactness; and that too, when *all other Methods* would prove wholly ineffectual.—It may be proper to observe farther, that neither of the foregoing Rules will (without taking a greater Number of equidistant Perpendiculars) approximate the Measures of such curvilineal Planes as above-mentioned, so near as that deduced from *Cor.* 1; provided the Measure of the Segment, or the Part bounded by the Curve and the least Ordinate, *&c.* can be *truly* (or *very nearly*) obtained: Because the Error increases towards the least Ordinates or Perpendiculars, on Account of their Obliquity to the Curve; the Length whereof, between any two adjacent Perpendiculars, keeps continually increasing, and consequently gives greater Latitude for different Curves to pass through the same three Points, in those equidistant Perpendiculars.

If the Number of equidistant Ordinates (or Perpendiculars) exceeds three; then the general Rule derived from *Cor.* 1. will not *strictly* agree with those Rules obtained from the general Method of Differences; which determines a Parabolic Curve of *some* Kind to pass through *any* assigned Number of Points: Whereas, in the Method here laid down, common Parabolic Curves are supposed to be described through the 1st, 2d, and 3d, the 3d, 4th, and 5th, and through the 5th, 6th, and 7th, *&c.* Ordinates; but the Difference arising from computing the Area of any curvilineal Plane, by these two Methods, is extremely small, and can never be of the least Consequence in the Business of Gauging, *&c.*

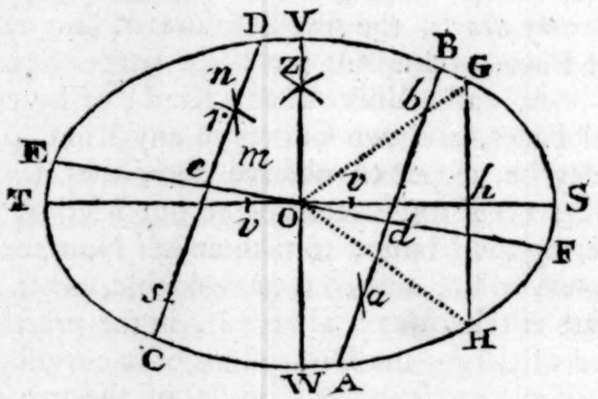
ſo near as the following *general Rule*; which determines, *very nearly*, the *true* Meaſure of any curvilineal Plane, bounded by three perpendicular Right-lines, and a Curve of any Kind; or by two parallel Lines, and two Curves of any Kind.

It may be proper to obſerve here, that it will be always neceſſary, according to our Method of conſidering the Matter, to take an *odd* Number of Ordinates; which indeed is unavoidable, when an Ordinate is taken (as it always is, in the practical Method of taking the Dimenſions of a curvilineal Back, &c.) *exactly* in the Middle of the two extreme Ordinates: And therefore this Method, which is comprehended in *one general Rule*, let the *odd* Number of Ordinates be what it will, claims the Preference in Point of Expedition; ſince it will appear, by the following Examples, that a large Number of Ordinates does but very little increaſe the Labour [redacted] Computation, which is far from of being the Caſe in any Rules (founded on indubitable Principles) I have hitherto met with.

Before we proceed to exemplify this Method, it may not be amiſs to give a few general Directions for taking the Dimenſions of curvilineal Backs; ſuch as are now frequently made Uſe of by *Diſtillers*, &c.

Let the Bottom of an *Elliptical Veſſel* be repreſented by the following Figure; then in Order to find the Center, and alſo to draw the Tranſverſe and Conjugate Diameters thereof, proceed as follows.

First,



First, with your Chalk-Line, in any Part of the Back, ſtrike a Line AB; then, with one Foot of your Compaſſes on *s*, at ſome convenient Diſtance from AB, deſcribe an Arch of a Circle, ſo as to cut that Line, as at *a*; and with the ſame Extent upon *b*, ſome Diſtance from *a* (in the Line AB) deſcribe the Arch *mn*; alſo, upon *s* as a Center, with the Extent *ab*, deſcribe another Arch, cutting *mn* in *r*; through the Points *s* and *r*, ſtrike (with your Chalk-Line) the Right-line CD, which is parallel to AB, and through *e* and *d*, the Middle of CD and AB, draw the Line EF; the Middle of which is (at O) the Center of the Ellipſis: Upon O, as a Center, with a Line (or String) of a ſuitable Length, let two Marks be made in the Periphery of the Figure, as at G and H, and mark the Right-line GH, which biſect (*i. e.* divide into two equal Parts) in the Point *h*; through which, and the Center O, ſtrike (with the Chalk-Line) the Tranſverſe Diameter TS; and upon the two Points *v* and *v*, as Centers, at a ſmall and equal Diſtance from O, with a large Extent of the Compaſſes, deſcribe two Arches interſecting each other in *z*; then through O and *z*, draw the Conjugate Diameter VW.

After

After the Bottom of the Veſſel is thus quartered, draw the Ordinates, or Right-lines, perpendicular to EF, by the following Method. (See the Figure *Pa.* 246).

On each Side the Center O, in the Tranſverſe Diameter EF, ſet off, with your Compaſſes, any two arbitrary Diſtances; that is, let *rs* be equal to *mn* (*i. e.* each about 8 or 9 Inches, more or leſs, as may appear neceſſary): Then with any convenient Diſtance, or Diſtances (*i. e.* ſuch, that will make the Diſtance of two parallel Lines ſomewhat leſs than what you judge the leaſt Ordinate will be taken), and upon the Points *m*, *n*, and *r*, *s*, as Centers, deſcribe Arches of Circles, cutting each other in the Points *a*, *a*, and *c*, *c*; through which Points ſtrike, with your Chalk-Line, the *parallel Lines* NT and PM, interſecting the Conjugate Diameter in the Points *d* and *f* reſpectively: From the Point *d* (or *f*) ſet off, with your Compaſſes, each Way in the Line NT, 2, 3, 4, &c. equal Diſtances, according as you intend to take 5, 7, 9, &c. Ordinates; and at the ſame Time obſerve, that the Diſtance (*eg* or SV) of the extreme Ordinates be taken (according to the Size of the Back), about 4, 5, 6, &c. Inches leſs than the Tranſverſe Diameter at the Top: Again, with the ſame Extent, ſet off, each Way from the Point *f*, the ſame Number of equal Diſtances as thoſe ſet off above in the Line NT; that is, let *fl*, *lk*, *fp* and *pv* be each taken equal to *db* (or *eb*, &c.); then, with the Chalk-Line, through the Points *e*, *v*; *b*, *p*; *t*, *l*; and *g*, *k*, mark the Ordinates (or Perpendiculars) AB, CD, GH, and IK.

Note. It is wholly immaterial whether or not the parallel Lines, PM and NT, are taken equidiſtant from the Tranſverſe Diameter; that is, it makes no Difference whether we deſcribe, on the Points *m*, *n*, and *r*, *s* (as Centers) the Arches of

Circles, on each Side the Tranſverſe, with one common Radius (or Interval), or with two different Radii (or Intervals).

Now, in Order to draw Lines from the Extremities of the Ordinates, *&c.* up the Sides of the Back (ſo that the Ordinates may be taken, in any Part of its Depth, at the ſame equal Diſtances as thoſe at the Bottom), proceed in the following Manner. Hold a *(chalked)* Plumb-Line at the Top of the Back, directly over the Ordinate AB; then, order your Aſſiſtant to hold the Plummet at the Extremity A, ſtrike a Line againſt the Side of the Back, from the Bottom to the Top: Proceed in the very ſame Manner at the other Points C, R, G, I, F, K, *&c.* to E.

A GENERAL RULE, *for determining*, very nearly, *with any odd Number of equidiſtant perpendicular Ordinates*, &c. *the Meaſure of any curvilineal Plane, bounded at each End by an Ordinate.*

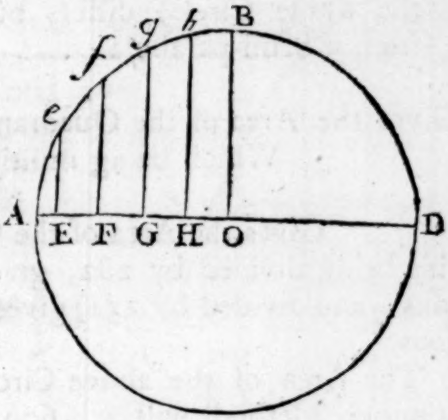
To the Sum of the firſt and laſt, or the two extreme Ordinates, add four times the Sum of the 2d, 4th, 6th, and 8th, *&c.* Ordinates, and alſo Double the Sum of all the Reſt; this Total being multiplied by one-third of the common Diſtance of the Ordinates; the Product will give the required Area, *very nearly:* Which being divided by 282, 231, and by 2150, the Quotients will be the Area in Ale and Wine Gallons, and Malt Buſhels reſpectively.

EXAMPLE I.

Let it be required to find the Area of a Circle, whoſe Diameter is 36 Inches.

In

In the Quadrant AOB, draw four equidistant Lines, perpendicular to the Diameter AD, and let their common Distance EF (FG, *&c.*) be 4 Inches; then, by the Property of the Circle, we get the Lengths of those Perpendiculars; *viz.*



E*e*	equal to	8.24
F*f*	=	13.41
G*g*	=	16.12
H*h*	=	17.55
OB	=	18.

Hence the following

Operation.

The Sum of the extreme Ordinates is 26.24

Four times the Sum of the 2d and 4th (30.96) is } 123.84

Twice the Middle Ordinate (16.12) is 32.24

Total 182.32

Which being multiplied by 4, and the Product divided by 3; or, which comes to the same, multiplied by one-third of the common Distance, gives the Area of the Space OB*e*E 243.093

Add the Segment AE*e*; which (by Reason of its Smallness, with Regard to the whole Circle) differs but little from a Semi-parabola . . . . } 10.98

Gives the Area of the Quadrant AOB 254.073

Which being multiplied by . . . 4

Gives the Area of the Circle 1016.292; this being divided by 282, gives 3.603 Ale Gallons; and divided by 231, gives 4.399 Wine Gallons.

The Area of the above Circle, found by the common Method, will be 3.609 Ale Gallons, and 4.406 Wine Gallons; which exceed the former but about 6 thousandths of an Ale Gallon, and 7 thousandths of a Wine Gallon: And by making Use of a greater Number of Ordinates, the above Differences would still have been less.

EXAMPLE 2.

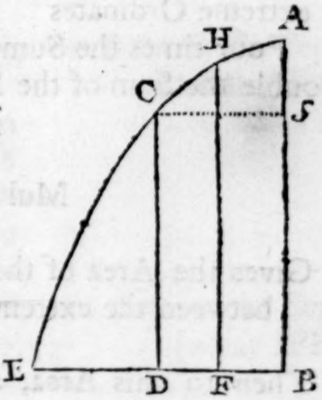
In a common Parabola, whose Abscissa AB is 32, and the Semi-ordinate FB 24; it is required to find the Area of the Part ABDC, when BD (or the Semi-ordinate C*s*) is equal to 12.

OPERATION.

First there is given AB=32; and, by the Property of the Curve, *Pa.* 65, we have FH=30, and

and alſo BD=24; whence, by the preceding *general Rule*,

The Sum of the two Extremes	56
Four times the Middle Perpendicular FH	120
Total	176
Multiplied by ⅓d of the common Diſtance DF (FB)	2
Gives the Area (*ſtrictly true*) of the Part ABDC	352.



For, to the Parabolic Area CA*s* (*ſee Pa.* 95) 64
Add the Area of the Rectangle DC*s*B 288

Gives the required Area 352, [*as before.*

EXAMPLE 3.

To find the Area of the curvilineal Plane ERFLE in Ale and Wine Gallons; whoſe Axis EF (biſected by RL) is ſuppoſed equal to 112 Inches, and alſo the perpendicular Ordinates, and their common Diſtance aſunder, as below.

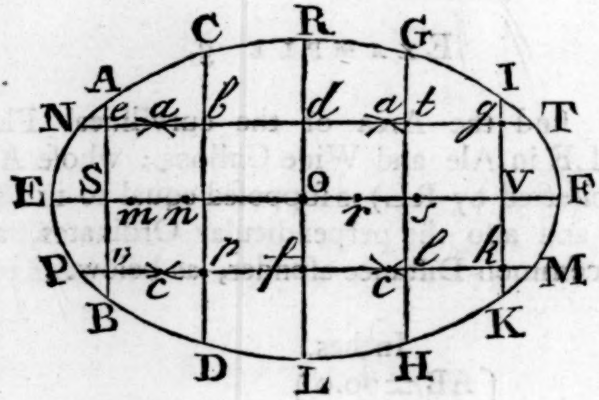
	Inches.	
Ordinates.	AB = 70.0	Their common Diſtance (*bd*, *&c.*) is 24 Inches; and therefore E*b* (or F*c*) is 8 Inches.
	CD = 79.0	
	RL = 80.0	
	GH = 78.6	
	IK = 69.0	

OPERATION.

OPERATION.

By the preceding *general Rule*, the Sum of the two extreme Ordinates	139
Four times the Sum of the 2d and 4th	630.4
Double the Sum of the Rest (*viz.* the 3d)	160
Total	929.4
Multiplied by $\frac{1}{3}$ of 24	= 8
Gives the Area of the Part contained between the extreme Ordinates	7435.2

Then to this Area, add that of the Segments AEB and IFK (which, on Account of the Smallness of ES or FV with Regard to AB or IK, is immaterial whether they are considered as Parabolas, or the Segments of Circles), and we shall then have the Area of the whole Figure ERFLE.



AB=70

AB = 70
Multiply by ES 8

Product 560

⅔ds whereof is 373.33, the Area of the Segment AEB.

Again IK = 69
Multiply by FV 8

Product 552

⅔ds of which is 368, the Area of the Segment IFK.
Add 373.33

The Area of both Segments taken as Parabolas 741.33
Add 7435.2

Whole Content in Inches . . 8176.53

282)8176.53(29, the Area in Ale Gallons.
231)8176.53(35.39, the Area in Wine Gallons.

Because, in Practice, the Middle Ordinate RL (see the preceding *Figure*) always bisects the Axis EF, consequently ES is equal to FV: Therefore the Area of the two Segments, AEB and IFK, may be more readily obtained by multiplying the Sum of the two extreme Ordinates, AB and IK, by ES or FV, and taking two-thirds of the Product: Or, which is the same Thing, multiplying the said Sum of the Ordinates by twice ES (or twice FV), and taking one-third of the Product: As in the following Operations.

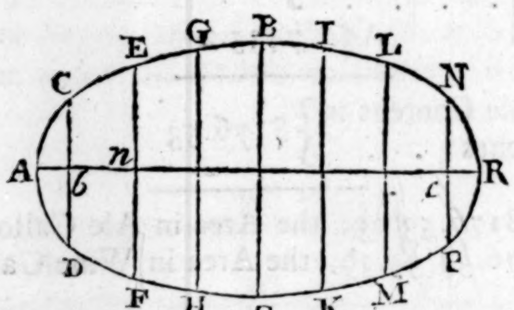
EXAMPLE

EXAMPLE 4.

Required the Area of the curvilineal Plane AB RS in Ale and Wine Gallons; when AR (bisected by BS) is equal to 102 Inches, the perpendicular Ordinates, and their common Distance as follow.

Perpendicular Ordinates	CD = 54.0 EF = 60.0 GH = 62.2 BS = 63.0 IK = 62.4 LM = 60.1 NP = 54.6	Their common Distance 15 Inches; and therefore A*b* (or R*c*) equal to 6 Inches.

OPERATION.



The Sum of the two extreme Ordinates	108.6
Four times the Sum of the 2d, 4th, and 6th Ordinates	732.4
Twice the Sum of the Rest (*i.e.* 3d and 5th)	249.2
Total	1090.2
Multiplied by one-third of 15; *viz* . . .	5
Gives the Area of the Part DCNP =	5451.0

FOR

FOR THE AREA OF THE TWO *SEGMENTS*.

The Sum of the first and last Ordinates	108.6
Multiply by twice A*b* (or twice R*c*) . .	12
Product	1303.2
One-third whereof is	434.4
Add the Area found above	5451
The whole Content in Inches	5885.4

282)5885.4(20.87=the Area in Ale Gallons.
231)5885.4(25.477 Wine Gallons.

The Area of the foregoing Figure, being computed by the Method laid down in *Shirtcliffe*'s Gauging, *Pa.* 187, will come out 20.862 and 25.468 Ale and Wine Gallons respectively.

But, if the Area of the said Figure be computed as an Ellipsis, it will come out 21.84 Wine Gallons; which is 3.63 Gallons *too little.*—Whence it is manifest, that the Revenue may be greatly injured, by gauging *all* curvilineal Vessels as *Ellipses:* But, if a due Regard be paid to the Method here laid down, we shall be always certain of obtaining, *very nearly*, the *true* Area of any curvilineal Vessel, let its Form be what it will.

Though it is here said, that the Revenue may be greatly injured, by gauging *all* curvilineal Vessels as *Ellipses*, yet I would not be understood to mean, that *every* curvilineal Vessel, now made use of by *Brewers*, *Distillers*, &c. is *greater* than an *Ellipsis* of the same Transverse and Conjugate Diameters: For, as a vast Variety of Curves may be described thro' the same four Points (A, B, C, D, see, for Instance, the following Figure), it may sometimes happen that such a *Vessel* may be *less*

than an Ellipsis; and, therefore, the *general Method* here given (which *always* approximates the *true* Measure of the Vessel, let its Form be ever so irregular) will, in such Case, be found serviceable to the Trader; as well as to the Revenue when the Vessel is *greater* than an Ellipsis.

It will be necessary to inform the *practical* Reader, that the Semi-ordinate (E*n* or F*n*) of an Ellipsis (see the preceding Figure) is always equal to the Square Root of the Product of the two Parts (A*n* and *n*R) of the Transverse corresponding to that Semi-ordinate, multiplied by the Quotient of the Conjugate (B~~D~~) divided by the Transverse Diameter (AR).
S

Hence it follows, that when the Semi-ordinates, thus obtained, come out greater than what appears from their *actual* Mensuration, the Vessel is then *less* than an Ellipsis: But, on the contrary, when the said Semi-ordinates come out, by the above Method of Computation, less than what is found by Mensuration (which will, I believe, most frequently occur in Practice); then will the Measure of such curvilineal Vessel be *greater* than that of an Ellipsis described through the same four Points (A, B, R, S, see the preceding Figure).— As, for Instance, in the preceding Example, BS is equal to 63, A*n*=21, and *n*R=81 Inches: Whence, by the above Rule, 81 being multiplied by 21, gives 1701, whose Square Root is 41.2: Also 63 (the Conjugate) being divided by 102 (the Transverse), the Quotient will be .61; then if 14.2 be multiplied by .61, the Product will be 25.132, the Semi-ordinate E*n* (or F*n*); which, by Mensuration, was found (see *Pa.* 248) to be 30 Inches.— Whence it appears by this Method, that the Measure of a curvilineal Vessel, having its Diameters equal (or in Proportion) to those of the

the 4th Example, will be confiderably greater than an Ellipfis, defcribed thro' the Extremities of the fame Tranfverfe and Conjugate Diameters.

Example 5.

Let the Tranfverfe and Conjugate Diameters of a curvilineal Veffel be *here* fuppofed the fame as in the laft *Example*, and let the equidiftant perpendicular Ordinates, &c. be as follow; required the Area thereof in the Wine Gallons.

Perpendicular Ordinates	CD = 30.4 EF = 40.5 GH = 48.2 BS = 63.0 IK = 48.4 LM = 40.2 NP = 30.0	Their common Diftance (as before) 15 Inches; and therefore A*b* or R*c*) equal to 6 Inches. (See the foregoing *Fig.*).

Operation.

The Sum of the two extreme Ordinates	60.4
Four times the Sum of the 2d, 4th, and 6th Ordinates	574.8
Double the Sum of all the Reft (*i. e.* the 3d, and 5th). . .	193.2
Total	828.4
Multiplied by $\frac{1}{3}$d of the common Diftance; *i. e.*	. . 5
Gives the Area of the Part comprehended between the extreme Ordinates	4142.0
Add the Segments, found by the preceding Method . . .	241.6
Gives the whole Area	4383.6 Inches;

Inches; which being divided by 231, the Quotient will be 18.97 Wine Gallons, the required Area, *nearly*: But, if this Veſſel be conſidered as an Ellipſis, its Area will then appear to be 21.84 W. Gallons; being *nearly* three Gallons more than the *true* Area.

EXAMPLE 6.

Wherein is it propoſed to find the Area of the curvilineal Space ABCDA (not *Elliptical*, but of an *unknown* Form), whoſe Dimenſions were obtained, by *actual* Menſuration, in the following Manner; *viz.*

The Axis AC (or twice A*a*) is equal to 151.7 Inches.

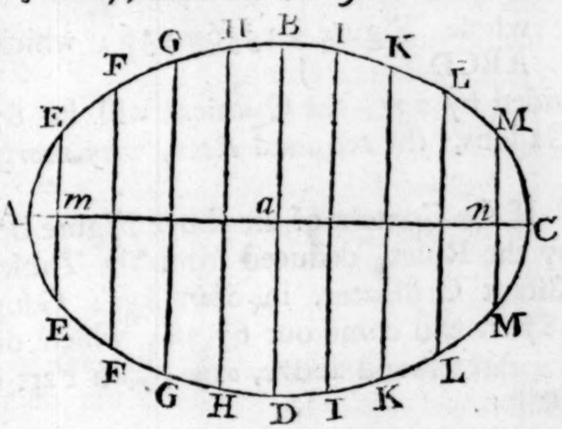
	EE	equal to 61,4	Their common Diſtance is 17 Inches; and conſequently A*m* (or C*n*) is 7.85 Inches.
Perpendicular Ordinates,	FF	= 101.2	
	GG	= 118.0	
	HH	= 123.7	
	BD	= 125.2	
See Overley's *Gauging, Pa.* 281.	II	= 124.0	
	KK	= 118.6	
	LL	= 99.9	
	MM	= 57.0	

OPERATION.

Operation.

By the foregoing *general Rule*, we have the Sum of the two extreme Ordinates . .	118.4
Four times the Sum of the 2d, 4th, 6th, and 8th Ordinates (beginning at either End) . . .	1795.2
Twice the Sum of all the Rest (*viz.* the 3d, 5th and 7th) . .	723.6
Total	2637.2
Which being multiplied by 17, and the Product divided by 3 (or multiplied by $\frac{1}{3}$ of 17) becomes . .	14944.13 =

the Area in Inches, of the Space contained between the extreme Ordinates; to this, add the Area of the two Segments EAE and MCM



(considered as Parabolas), and we shall then obtain the Area of the whole curvilineal Space ABCD, *very nearly:* See the Remainder of the Operation.

The

The Sum of the two extreme Ordinates }	118.4
Multiply by Am (or Cn)	7.85
	5920
	9472
	8288
Product	929.440

$\frac{2}{3}$ds whereof is 619.626 = the Area of both [Segments.

Add the Area of the Space found above . . }	14944.13
The Area of the whole Figure ABCDA }	15563.756 ; which being di-

vided by 231, the Quotient will be 67.37 Wine Gallons, the required Area, *very nearly.*

If the Content of the above Figure be computed by the Rules, deduced from the Tables of equidistant Ordinates, in *Shirtcliffe*'s Gauging, *Pa.* 187, it will come out 67.38 ; which differs from the Area found above, *only* $\frac{1}{100}$th Part of a Wine Gallon.

Note. It may be proper to take Notice, that, according to this Method of Computation, it will be the most commodious to write down the Dimensions, *&c.* of a curvilineal Vessel, in the following Manner.

Mr.

Mr. ——'s 5th Back, gauged *Nov.* 22, 1764.

ORDINATES 13 Inches equidistant.										
Inches.	Transverse.	1	2	3	Conjugate 4	5	6	7	Area.	Gallons
13	84.8	29.3	50.7	57.5	58.4	56.8	49.7	24.7	17.74	230.62
11	86.0	31.5	51.6	58.1	59.0	57.5	50.8	27.0	18.22	200.42
11	86.9	33.0	52.3	58.8	59.7	58.3	52.0	29.2	18.66	205.26
11	87.9	34.5	53.1	59.3	60.3	58.9	53.0	31.5	19.10	210.10
10	88.8	36.0	53.8	60.0	60.8	59.6	53.8	33.7	19.51	195.10
1.3	*Drip.* . . .									20.
57.3	*Depth.*								*Content*	1061.50

Depth 57.3 Gall.
Drip 1.3 . . . 20

Neat Depth 56.0

Before I quit this Subject, it may not be amiss to give one Example more, in Order to shew the Method of computing the Area of a curvilineal Plane, by a Rule (for 13 equidistant Ordinates) deduced from the Tables laid down in *Shirtcliffe's* Gauging, *Pa.* 187; and then to give the Operation by the *general Rule*, *Pa.* 242; whereby, I apprehend, its Utility will manifestly appear to every impartial Reader.

RULE.

The Sum of the 1st, 13th, and twice the (7th) Middle Ordinate multiplied by 41; the Sum of the 2d, 6th, 8th and 12th Ordinates multiplied by 216; the Sum of the 3d, 5th, 9th, and 11th Ordinates multiplied by 27; and the Sum of the 4th and 10th multiplied by 272: Then the Sum of these four Products being multiplied by the Distance of the two extreme Ordinates, and that Product divided by 1680, the Quotient will be the Area (in Inches) of any curvilineal Space contained between the extreme Perpendiculars.

EXAMPLE

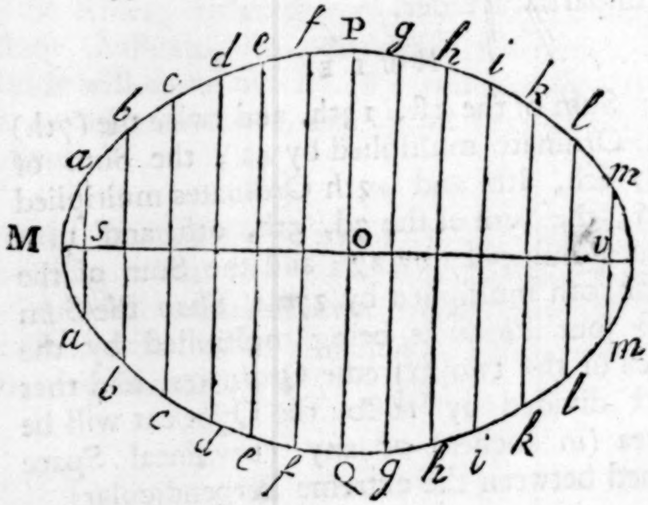
EXAMPLE 7.

To find the Area of the curvilineal Plane MPRQ in Wine Gallons; when MR (or twice MO) is equal to 148 Inches, the perpendicular Ordinates, and their common Diſtance as below.

Perpendicular Ordinates

aa	=	82.4
bb	=	96.5
cc	=	112.0
dd	=	119.2
ee	=	121.3
ff	=	122.4
PQ	=	123.0
gg	=	122.6
hh	=	121.2
ii	=	118.9
kk	=	112.0
ll	=	96.3
mm	=	82.4

Their common Diſtance is 12 Inches; and therefore M*s* (or R*v*) is 2 Inches.



OPERATION.

OPERATION, *by the preceding Rule.*

The 1st, 13th, and twice the Middle Ordinate	82.4
	82.4
	246.0
Sum	410.8
Multiplied by	41
	4108
	16432
1st Product	16842.8

The 2d, 6th, 8th, and 12th Ordinates	96.5
	122.4
	122.6
	96.3
Sum	437.8
Multiplied by	216
	26268
	4378
	8756
2d Product	94564.8

The 3d, 5th, 9th and 11th Ordinates	112.0
	121.3
	121.2
	112.0
Sum	466.5
	27
	32655
	9330
3d Product	12595.5
The 4th and 10th Ordinates	119.2
	118.9
Sum	238.1
Multiplied by	272
	4762
	16667
	4762
4th Product	64763.2
3d Product	12595.5
2d Product	94564.8
1st Product	16842.8
The Sum of the four Products	188766.3
Multiplied by the Distance of the extreme Ordinates . . .	144
	7550652
	7550652
	1887663
Gives	27182347.2
	1680)

```
1680)27182347.20(16179.96
     1680              219.73 = the Area of the 2
     ----              ------[Seg. taken as Parab.
     10382            16399.69 = the whole Con-
     10080                      [tent in Inches.
     -----
      3023
      1680
     -----
     13434
     11760
     -----
     16747
     15120
     -----
     16272
     15120
     -----
      11520
      10080
      -----
       1440
```

231)16399.69(70.99 Wine Gallons; the required Area, *nearly*.

OPERATION, by the *general Rule*, Pa. 242.

The Sum of the extreme Ordinates is 164.8

The 2d, 4th, 6th, 8th, 10th, and 12th Ordinates $\left\{\begin{array}{r} 96.5 \\ 119.2 \\ 122.4 \\ 122.6 \\ 118.9 \\ 96.3 \end{array}\right.$

Sum 675.9 multiplied by 4, gives [2703.6.

 The

The 3d, 5th, 7th, 9th, and 11th Ordinates		112.0
		121.3
		123.0
		121.2
		112.0
	Sum	589.5

Which multiplied by 2, gives 1179.0; to which

164.8 and 2703.6 (found above) being added, gives	4047.4
Multiplied by $\frac{1}{3}$d of 12	4
Gives	16189.6
The Area of the Segments	219.73

231)16409.33(71.03 W. Gallons.

The Areas obtained, by these two Methods, differ but about $\frac{4}{100}$th Part of a Wine Gallon; and both the Operations were given at Length, in Order to shew the vast Advantage which the latter has over the former.

☞ It is manifest (by the Writers on Fluxions) that the Radius of Curvature of a *Parabola* is infinite, at an infinite Distance from its Vertex; therefore the Difference between a Right-line and a Parabolic Curve (of any Kind), at an infinite Distance from the Vertex, is less than any assignable Quantity; and consequently, as the Methods here proposed (or those Rules derived from the general Principles) are *strictly true*, in *every* Part of a *Parabolic Curve*, it evidently follows, that, at an infinite Distance from the Vertex, the *Measure* of a Space, obtained by any of these Rules, will differ from *that*, when considered as a right-lined Space, by a Quantity less than any given, or assignable, Quantity whatever: But (by *Lemma* 1st, *Pa.* 99, of *De L'Hospital*'s Conic Sections) *if the Difference of two Quantities does continually diminish, so that at last it becomes less than any given Quantity; then will those Quantities at last be equal.* — Hence it is evident, that by these Methods we can obtain the *true* (and not the *approximate*) Measure of any rectilineal Plane whatever: For any right-lined Plane may be divided into Quadrilaterals (whereof two Sides must be parallel) and Triangles; then may the *true* Measures of those Figures be separately found, by any assigned Number of equidistant Perpendiculars whatever. — Or, the *true* Area may be obtained, by dividing the Figure into Triangles.

Suppose,

Suppofe, for Example, it was required to find the Meafure of the right-lined Space ABED, when AB (*Fig.* XVII. is parallel to DE) = 20, DE = 12, and the Perpendicular DF = 30 Inches.

First, *by three equidiftant Perpendiculars.*

The Sum of the Extremes	32
Four times the Middle Perpendicular (16) . .	64
Total	96
Multiplied by $\frac{1}{3}$d of 15 . .	5
Gives the required Meafure	480 = CC

$$\left(\frac{AB+DE}{2}\right) \times DF = 16 \times 30$$; which is well known from other Principles.

By five equidiftant Perpendiculars.

If DP (*Fig.* XVII.) be drawn parallel to EB, and GG, CC and II be drawn parallel to AB, equidiftant from one another, and the other Dimenfions the fame as above ; then it is evident that $Id=2$, $Cc=4$, and $Gb=6$; whence DE = 12, II = 14, CC = 16, GG = 18, and AB = 20 : Then by the *general Rule, Pa.* 242.

The Sum of the two extreme Perpendiculars	32
Four times the Sum of the 2d and 4th Perpendiculars	128
Twice the Sum of the Reft (*viz.* the 3d)	32
Total	192
Multiplied by $\frac{1}{3}$d of the common Diftance (7.5) . .	2.5
	960
	384
Gives the Content as before	480.0.

Suppofe, in the Triangle ADP (*Fig.* XVII.) AP = 8, DF = 30 ; which being divided into eight equal Parts, and Lines drawn parallel to the Bafe, we fhall have $aa=1$, $Id=2$, $nm=3$, $Cc=4$, $rr=5$, $Gb=6$, $ee=7$, and AP = 8 : Whence, by *Cor.* 2, *Pa.* 237, we get $\overline{1+3+5+7} \times 4 + 8 + \overline{2+4+6} \times 2 \times \frac{3.75}{5} = 120\ (= 30 \times 4)$ = the Area of the Triangle ADP.

A TABLE *of the Areas of Circles in* ALE GALLONS, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 216 *Inches.*

Dia. in Inc.	0	.1	.2	.3	.4
1	0.0027	.0033	0.0040	0.0047	0.0054
2	0.0111	0.0122	0.0134	0.0147	0.0160
3	0.0250	0.0267	0.0285	0.0303	0.0321
4	0.0445	0.0468	0.0491	0.0514	0.0539
5	0.0696	0.0724	0.0753	0.0782	0.0812
6	0.1002	0.1036	0.1070	0.1105	0.1140
7	0.1364	0.1403	0.1443	0.1484	0.1525
8	0.1782	0.1827	0.1872	0.1918	0.1965
9	0.2255	0.2306	0.2357	0.2408	0.2460
10	0.2785	0.2841	0.2897	0.2954	0.3012
11	0.3369	0.3431	0.3493	0.3556	0.3619
12	0.4010	0.4077	0.4145	0.4213	0.4282
13	0.4706	0.4779	0.4852	0.4926	0.5000
14	0.5458	0.5537	0.5615	0.5695	0.5775
15	0.6266	0.6350	0.6434	0.6519	0.6605
16	0.7129	0.7219	0.7309	0.7399	0.7490
17	0.8048	0.8143	0.8239	0.8335	0.8432
18	0.9023	0.9124	0.9225	0.9327	0.9429
19	1.0054	1.0160	1.0266	1.0374	1.0482
20	1.1140	1.1252	1.1364	1.1477	1.1590
21	1.2282	1.2399	1.2517	1.2635	1.2754
22	1.3479	1.3602	1.3726	1.3850	1.3974
23	1.4733	1.4861	1.4990	1.5120	1.5250
24	1.6042	1.6176	1.6310	1.6445	1.6581
25	1.7406	1.7546	1.7686	1.7827	1.7968
26	1.8827	1.8972	1.9118	1.9264	1.9411
27	2.0303	2.0454	2.0605	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2305	2.2463
29	2.3422	2.3584	2.3746	2.3909	2.4073

A

A TABLE *of the Areas of Circles in* ALE GALLONS, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 216 *Inches.*

Dia. in Inc.	.5	.6	.7	.8	.9
1	0.0062	0.0071	0.0080	0.0090	0.0100
2	0.0174	0.0188	0.0203	0.0218	0.0234
3	0.0341	0.0360	0.0381	0.0402	0.0423
4	0.0563	0.0589	0.0615	0.0641	0.0668
5	0.0842	0.0873	0.0904	0.0936	0.0969
6	0.1176	0.1213	0.1250	0.1287	0.1325
7	0.1566	0.1608	0.1651	0.1694	0.1738
8	0.2012	0.2059	0.2[illegible]08	0.2156	0.2206
9	0.2513	0.2566	0.2620	0.2674	0.2729
10	0.3070	0.3129	0.3188	0.3248	0.3308
11	0.3683	0.3747	0.3812	0.3877	0.3943
12	0.4351	0.4421	0.4492	0.4563	0.46[illegible]4
13	0.5075	0.5151	0.5227	0.5303	0.5381
14	0.5855	0.5936	0.6018	0.6100	0.6183
15	0.6691	0.6777	0.6864	0.6952	0.7041
16	0.7582	0.7674	0.7767	0.7860	0.7954
17	0.8529	0.8627	0.8725	0.8824	0.8923
18	0.9532	0.9635	0.9739	0.9843	0.9948
19	1.0590	1.0699	1.0808	1.0918	1.1029
20	1.1704	1.1818	1.1933	1.2049	1.2165
21	1.2874	1.2994	1.3114	1.3235	1.3357
22	1.4099	1.4225	1.4351	1.4478	1.4605
23	1.5380	1.5511	1.5643	1.5775	1.5908
24	1.6717	1.6854	1.6991	1.7129	1.7267
25	1.8110	1.8252	1.8395	1.8538	1.8682
26	1.9558	1.9706	1.9854	2.0003	2.0153
27	2.1062	2.1215	2.1369	2.1524	2.1679
28	2.2621	2.2781	2.2940	2.3100	2.3261
29	2.4237	2.4401	2.4567	2.4732	2.4899

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	0	.1	.2	.3	.4
30	2.5065	2.5233	2.5401	2.5569	2.5738
31	2.6764	2.6937	2.7111	2.7285	2.7459
32	2.8519	2.8697	2.8877	2.9056	2.9236
33	3.0329	3.0513	3.0698	3.0883	3.1069
34	3.2195	3.2385	3.2575	3.2766	3.2957
35	3.4117	3.4312	3.4508	3.4704	3.4901
36	3.6094	3.6295	3.6497	3.6698	3.6901
37	3.8128	3.8334	3.8541	3.8748	3.8956
38	4.0216	4.0428	4.0641	4.0854	4.1067
39	4.2361	4.2578	4.2796	4.3015	4.3234
40	4.4561	4.4784	4.5008	4.5232	4.5457
41	4.6817	4.7046	4.7275	4.7505	4.7735
42	4.9129	4.9363	4.9598	4.9833	5.0069
43	5.1496	5.1736	5.1976	5.2217	5.2459
44	5.3919	5.4164	5.4410	5.4657	5.4904
45	5.6398	5.6649	5.6900	5.7152	5.7405
46	5.8932	5.9189	5.9446	5.9703	5.9962
47	6.1522	6.1784	6.2047	6.2310	6.2574
48	6.4168	6.4436	6.4704	6.4973	6.5242
49	6.6870	6.7143	6.7417	6.7691	6.7966
50	6.9627	6.9906	7.0185	7.0465	7.0745
51	7.2440	7.2724	7.3009	7.3295	7.3581
52	7.5309	7.5599	7.5889	7.6180	7.6472
53	7.8233	7.8528	7.8825	7.9121	7.9418
54	8.1213	8.1514	8.1816	8.2118	8.2421
55	8.4249	8.4555	8.4863	8.5170	8.5479
56	8.7340	8.7652	8.7965	8.8279	8.8592
57	9.0487	9.0805	9.1124	9.1442	9.1762
58	9.3690	9.4014	9.4338	9.4662	9.4987
59	9.6949	9.7278	9.7607	9.7937	9.8268
60	10.0263	10.0598	10.0933	10.1268	10.1604

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	5.	.6	.7	.8	.9
30	2.5908	2.6078	2.0249	2.6420	2.6592
31	2.7035	2.7810	2.7987	2.8164	2.8341
32	2.9417	2.9598	2.9780	2.9963	3.0146
33	3.1255	3.1442	3.1630	3.1818	3.2006
34	3.3149	3.3342	3.3535	3.3728	3.3922
35	3.5099	3.5297	3.5495	3.5694	3.5894
36	3.7104	3.7308	3.7512	3.7716	3.7922
37	3.9165	3.9374	3.9584	3.9794	4.0005
38	4.1282	4.1496	4.1712	4.1928	4.2144
39	4.3454	4.3674	4.3895	4.4117	4.4339
40	4.5682	4.5908	4.6134	4.6361	4.6589
41	4.7966	4.8197	4.8429	4.8662	4.8895
42	5.0305	5.0542	5.0780	5.1018	5.1257
43	5.2701	5.2943	5.3186	5.3430	5.3674
44	5.5151	5.5400	5.5648	5.5898	5.6147
45	5.7658	5.7912	5.8166	5.8421	5.8676
46	6.0220	6.0480	6.0739	6.1000	6.1261
47	6.2838	6.3103	6.3369	6.3635	6.3901
48	6.5512	6.5782	6.6053	6.6325	6.6597
49	6.8241	6.8517	6.8794	6.9071	6.9349
50	7.1027	7.1308	7.1590	7.1873	7.2156
51	7.3867	7.4154	7.4442	7.4730	7.5019
52	7.6764	7.7057	7.7350	7.7644	7.7938
53	7.9716	8.0014	8.0313	8.0613	8.0913
54	8.2724	8.3028	8.3332	8.3637	8.3943
55	8.5788	8.6097	8.6407	8.6717	8.7029
56	8.8907	8.9222	8.9537	8.9854	9.0170
57	9.2082	9.2402	9.2724	9.3045	9.3367
58	9.5313	9.5639	9.5965	9.6293	9.6620
59	9.8599	9.8931	9.9263	9.9596	9.9929
60	10.1941	10.2278	10.2616	10.2955	10.3294

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
61	10.3633	10.3973	10.4314	10.4655	10.4997
62	10.7059	10.7404	10.7751	10.8097	10.8445
63	11.0540	11.0891	11.1243	11.1595	11.1948
64	11.4077	11.4434	11.4791	11.5149	11.5508
65	11.7670	11.8032	11.8395	11.8759	11.9123
66	12.1318	12.1686	12.2055	12.2424	12.2793
67	12.5023	12.5396	12.5770	12.6145	12.6520
68	12.8783	12.9162	12.9541	12.9921	13.0302
69	13.2598	13.2983	13.3368	13.3754	13.4140
70	13.6469	3.6860	13.7250	13.7642	13.8034
71	14.0396	14.0792	14.1188	14.1585	14.1983
72	14.4379	14.4780	14.5182	14.5585	14.5988
73	14.8417	14.8824	14.9232	14.9640	15.0048
74	15.2512	15.2924	15.3337	15.3751	15.4165
75	15.6661	15.7079	15.7498	15.7917	15.8337
76	16.0867	16.1290	16.1715	16.2139	16.2565
77	16.5128	16.5557	16.5987	16.6417	16.6848
78	16.9445	6.9880	17.0315	17.0751	17.1187
79	17.3818	17.4258	17.4699	17.5140	17.5582
80	17.8246	17.8692	17.9138	17.9585	18.0033
81	18.2730	18.3181	18.3633	18.4086	18.4539
82	18.7270	18.7727	18.8184	18.8642	18.9101
83	19.1865	19.2328	19.2791	19.3255	19.3719
84	19.6516	19.6984	19.7453	19.7922	19.8392
85	20.1223	20.1697	20.2171	20.2646	20.3121
86	20.5985	20.6465	20.6945	20.7425	20.7906
87	21.0804	21.1289	21.1774	21.2260	21.2747
88	21.5678	21.6168	21.6659	21.7151	21.7643
89	22.0607	22.1103	22.1600	22.2097	22.2595
90	22.5593	22.6094	22.6596	22.7099	22.7602
91	23.0634	23.1141	23.1649	23.2157	23.2666

The

The Areas of Circles in Ale Gallons.

Dia. in Inc.	.5	.6	.7	.8	.9
61	10.5339	10.5682	10.6025	10.6369	10.6714
62	10.8792	10.9141	10.9490	10.9839	11.0189
63	11.2302	11.2656	11.3010	11.3365	11.3721
64	11.5867	11.6226	11.6586	11.6947	11.7308
65	11.9487	11.9852	12.0218	12.0584	12.0951
66	12.3164	12.3534	12.3906	12.4277	12.4650
67	12.6896	12.7272	12.7649	12.8026	12.8404
68	13.0683	13.1065	13.1448	13.1831	13.2214
69	13.4527	13.4914	13.5302	13.5691	13.6080
70	13.8426	13.8819	13.9212	13.9607	14.0001
71	14.2381	14.2779	14.3178	14.3578	14.3978
72	14.6391	14.6795	14.7200	14.7605	14 8011
73	15.0458	15.0867	15.1277	15.1688	15.2100
74	15.4580	15.4995	15.5411	15.5827	15.6244
75	15.8757	15.9178	15.9599	16.0021	16.0444
76	16.2991	16.3417	16.3844	16.4271	16.4699
77	16.7280	16.7712	16.8144	16.8577	16.9011
78	17.1624	17.2062	17.2500	17.2939	17.3378
79	17.6025	17.6468	17.6912	17.7356	17.7801
80	18.0481	18.0930	18.1379	18.1829	18.2279
81	18.4993	18.5447	18.5902	18.6357	18.6813
82	18.9560	19.0020	19 0481	19.0941	19.1403
83	19.4184	19.4649	19.5115	19.5581	19.6049
84	19.8863	19.9334	19.9805	20.0277	20.0750
85	20.3597	20.4074	20.4551	20.5029	20.5507
86	20.8388	20.8870	20.9352	20.9836	21.0319
87	21.3234	21.3721	21.4210	21.4698	21 5188
88	21.8135	21.8629	21.9123	21.9617	22.0112
89	22.3093	22.3592	22.4091	22.4591	22.5092
90	22.8106	22.8611	22.9115	22.9621	23.0127
91	23 3175	23.3685	23.4195	23.4707	23.5218

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
92	23.5730	23.6243	23.6756	23.7270	23.7785
93	24.0883	24.1401	24.1920	24.2439	24.2959
94	24.6091	24.6615	24.7139	24.7664	24.8190
95	25.1355	25.1884	25.2414	25.2945	25.3476
96	25.6674	25.7209	25.7745	25.8281	25.8818
97	26.2050	26.2590	26.3131	26.3673	26.4215
98	26.7481	26.8027	26.8573	26.9121	26.9668
99	27.2967	27.3519	27.4071	27.4624	27.5177
100	27.8510	27.9067	27.9625	28.0183	28.0742
101	28.4108	28.4670	28.5234	28.5798	28.6362
102	28.9761	29.0330	29.0899	29.1468	29.2038
103	29.5471	29.6045	29.6619	29.7194	29.7770
104	30.1236	30.1815	30.2396	30.2976	30.3558
105	30.7057	30.7642	30.8228	30.8814	30.9401
106	31.2933	31.3524	31.4115	31.4707	31.5300
107	31.8866	31.9462	32.0059	32.0656	32.1254
108	32.4854	32.5455	32.6058	32.6661	32.7264
109	33.0897	33.1505	33.2113	33.2721	33.3330
110	33.6997	33.7610	33.8223	33.8837	33.9452
111	34.3152	34.3770	34.4389	34.5009	34.5629
112	34.9362	34.9987	35.0611	35.1237	35.1862
113	35.5629	35.6259	35.6889	35.7520	35.8151
114	36.1951	36.2586	36.3222	36.3859	36.4496
115	36.8329	36.8970	36.9611	37.0253	37.0896
116	37.4763	37.5409	37.6056	37.6703	37.7352
117	38.1252	38.1904	38.2556	38.3209	38.3863
118	38.7797	38.8454	38.9113	38.9771	39.0430
119	39.4398	39.5061	39.5724	39.0389	39.7053
120	40.1054	40.1723	40.2392	40.3062	40.3732

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	·5	.6	·7	.8	·9
92	23.8300	23.8815	23.9331	23.9848	24.0365
93	24.3480	24.4001	24.4523	24.5045	24.5568
94	24.8716	24.9243	24.9770	25.0298	25.0826
95	25.4008	25.4540	25.5073	25.5606	25.6140
96	25.9355	25.9893	26.0431	26.0970	26.1510
97	26.4758	26.5301	26.5845	26.6390	26.6935
98	27.0217	27.0766	27.1315	27.1865	27.2416
99	27.5731	27.6286	27.6841	27.7397	27.7953
100	28.1302	28.1862	28.2422	28.2983	28.3545
101	28.6927	28.7493	28.8059	28.8626	28.9193
102	29.2609	29.3180	29.3752	29.4324	29.4897
103	29.8346	29.8923	29.9501	30.0078	30.0657
104	30.4139	30.4722	30.5305	30.5888	30.6472
105	30.9988	31.0576	31.1165	31.1754	31.2343
106	31.5893	31.6486	31.7080	31.7675	31.8270
107	32.1853	32.2452	32.3051	32.3652	32.4252
108	32.7868	32.8473	32.9078	32.9684	33.0290
109	33.3940	33.4550	33.5161	33.5772	33.6384
110	34.0067	34.0683	34.1299	34.1916	34.2534
111	34.6250	34.6871	34.7493	34.8116	34.8739
112	35.2489	35.3116	35.3743	35.4371	35.5000
113	35.8783	35.9416	36.0049	36.0682	36.1316
114	36.5133	36.5771	36.6410	36.7049	36.7689
115	37.1539	37.2182	37.2827	37.3471	37.4117
116	37.8009 2	37.8649	37.9299	37.9950	38.0600
117	38.4517	38.5172	38.5827	38.6483	38.7140
118	39.1090	39.1751	39.2411	39.3073	39.3735
119	39.7719	39.8385	39.9051	39.9718	40.0386
120	40.4403	40.5074	40.5747	40.6419	40.7092

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
121	40.7766	40.8440	40.9115	40.9790	41.0466
122	41.4534	41.5214	41.5894	41.6575	41.7256
123	42.1357	42.2043	42.2729	42.3415	42.4102
124	42.8236	42.8927	42.9619	43.0311	43.1004
125	43.5171	43.5868	43.6565	43.7263	43.7961
126	44.2162	44.2864	44.3567	44.4270	44.4974
127	44.9208	44.9916	45.0624	45.1333	45.2042
128	45.6310	45.7024	45.7737	45.8452	45.9167
129	46.3468	46.4187	46.4906	46.5626	46.6347
130	47.0681	47.1406	47.2131	47.2856	47.3582
131	47.7951	47.8680	47.9411	48.0142	48.0874
132	48.5275	48.6011	48.6747	48.7484	48.8221
133	49.2656	49.3397	49.4139	49.4881	49.5624
134	50.0092	50.0839	50.1586	50.2334	50.3082
135	50.7584	50.8336	50.9089	50.9842	51.0596
136	51.5132	51.5889	51.6648	51.7407	51.8166
137	52.2735	52.3498	52.4262	52.5027	52.5792
138	53.0394	53.1163	53.1932	53.2703	53.3473
139	53.8109	53.8883	53.9658	54.0434	54.1210
140	54.5879	54.6659	54.7440	54.8221	54.9003
141	55.3705	55.4491	55.5277	55.6064	55.6851
142	56.1587	56.2378	56.3170	56.3962	56.4755
143	56.9525	57.0321	57.1119	57.1917	57.2715
144	57.7518	57.8320	57.9123	57.9927	58.0731
145	58.5567	58.6375	58.7183	58.7992	58.8802
146	59.3671	59.4485	59.5299	59.6114	59.6929
147	60.1832	60.2651	60.3471	60.4291	60.5111
148	61.0048	61.0872	61.1698	61.2523	61.3350
149	61.8320	61.9150	61.9981	62.0812	62.1644
150	62.6647	62.7483	62.8319	62.9156	62.9994
151	63.5030	63.5872	63.6713	63.7556	63.8399

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	·5	·6	·7	·8	·9
121	41.1143	41.1820	41.2498	41.3176	41.3854
122	41.7939	41.8621	41.9304	41.9988	42.0672
123	42.4790	42.5478	42.6167	42.6856	42.7546
124	43.1697	43.2391	43.3085	43.3780	43.4475
125	43.8660	43.9359	44.0059	44.0759	44.1460
126	44.5678	44.6383	44.7089	44.7795	44.8501
127	45.2752	45.3463	45.4174	45.4885	45.5598
128	45.9882	46.0598	46.1315	46.2032	46.2750
129	46.7068	46.7789	46.8512	46.9234	46.9958
130	47.4309	47.5036	47.5764	47.6492	47.7221
131	48.1606	48.2339	48.3072	48.3806	48.4540
132	48.8959	48.9697	49.0436	49.1175	49.1915
133	49.6367	49.7111	49.7855	49.8600	49.9346
134	50.3831	50.4581	50.5331	50.6081	50.6832
135	51.1351	51.2106	51.2861	51.3618	51.4374
136	51.8926	51.9687	52.0448	52.1210	52.1972
137	52.6557	52.7324	52.8090	52.8858	52.9626
138	53.4244	53.5016	53.5788	53.6561	53.7335
139	54.1987	54.2764	54.3542	54.4321	54.5100
140	54.9785	55.0568	55.1352	55.2136	55.2920
141	55.7639	55.8428	55.9217	56.0006	56.0796
142	56.5549	56.6343	56.7137	56.7933	56.8728
143	57.3514	57.4314	57.5114	57.5915	57.6716
144	58.1535	58.2341	58.3146	58.3953	58.4759
145	58.9612	59.0423	59.1234	59.2046	59.2858
146	59.7745	59.8561	59.9378	60.0195	60.1013
147	60.5933	60.6755	60.7577	60.8400	60.9224
148	61.4177	61.5004	61.5832	61.6661	61.7490
149	62.2476	62.3309	62.4143	62.4977	62.5812
150	63.0832	63.1670	63.2509	63.3349	63.4189
151	63.9243	64.0087	64.0931	64.1777	64.2623

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
152	64.3469	64.4316	64.5163	64.6012	64.6860
153	65.1964	65.2816	65.3669	65.4523	65.5377
154	66.0514	66.1372	66.2231	66.3090	66.3950
155	66.9120	66.9983	67.0848	67.1712	67.2578
156	67.7781	67.8651	67.9520	68.0391	68.1262
157	68.6499	68.7374	68.8249	68.9125	69.0001
158	69.5272	69.6152	69.7033	69.7915	69.8797
159	70.4101	70.4987	70.5873	70.6760	70.7648
160	71.2985	71.3877	71.4769	71.5661	71.6554
161	72.1925	72.2822	72.3720	72.4618	72.5517
162	73.0921	73.1824	73.2727	73.3631	73.4535
163	73.9973	74.0881	74.1790	74.2699	74.3609
164	74.9080	74.9994	75.0908	75.1823	75.2739
165	75.8243	75.9162	76.0082	76.1003	76.1924
166	76.7462	76.8387	76.9312	77.0238	77.1165
167	77.6736	77.7667	77.8598	77.9529	78.0461
168	78.6066	78.7002	78.7939	78.8876	78.9814
169	79.5452	79.6394	79.7336	79.8279	79.9222
170	80.4893	80.5841	80.6788	80.7737	80.8686
171	81.4391	81.5343	81.6297	81.7251	81.8205
172	82.3943	82.4902	82.5861	82.6820	82.7780
173	83.3552	83.4516	83 5480	83.6446	83.7411
174	84.3216	84.4186	84.5156	84.6127	84.7098
175	85.2936	85.3911	85.4887	85.5863	85.6840
176	86.2712	86.3693	86.4674	86.5656	86.6638
177	87.2543	87.3530	87.4516	87.5504	87.6492
178	88.2431	88.3422	88.4415	88.5408	88.6401
179	89.2373	89 3371	89.4369	89.5367	89.6366
180	90.2372	90.3375	90.4378	90.5382	90.6387
181	91.2426	91.3435	91.4444	91.5453	91.6463
182	92 2536	92.3550	92.4565	92.5580	92.6596

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.5	.6	.7	.8	.9
152	64.7709	64.8559	64.9409	65.0260	65.1112
153	65.6232	65.7087	65.7943	65.8799	65.9656
154	66.4810	66.5671	66.6532	66.7394	66.8257
155	67.3444	67.4310	67.5177	67.6045	67.6913
156	68.2133	68.3005	68.3878	68.4751	68.5625
157	69.0878	69.1756	69.2634	69.3513	69.4392
158	69.9679	70.0562	70.1446	70.2330	70.3215
159	70.8536	70.9425	71.0314	71.1204	71.2094
160	71.7448	71.8343	71.9237	72.0133	72.1029
161	72.6416	72.7316	72.8217	72.9118	73.0019
162	73.5440	73.6345	73.7251	73.8158	73.9065
163	74.4519	74.5430	74.6342	74.7254	74.8167
164	75.3655	75.4571	75.5488	75.6406	75.7324
165	76.2845	76.3767	76.4690	76.5613	76.6537
166	77.2092	77.3020	77.3948	77.4877	77.5806
167	78.1394	78.2327	78.3261	78.4196	78.5131
168	79.0752	79.1691	79.2630	79.3570	79.4511
169	80.0166	80.1110	80.2055	80.3001	80.3947
170	80.9635	81.0585	81.1536	81.2487	81.3438
171	81.9160	82.0116	82.1072	82.2028	82.2986
172	82.8741	82.9702	83.0664	83.1626	83.2589
173	83.8377	83.9344	84.0311	84.1279	84.2247
174	84.8069	84.9042	85.0015	85.0988	85.1962
175	85.7817	85.8795	85.9774	86.0752	86.1732
176	86.7621	86.8604	86.9588	87.0573	87.1558
177	87.7480	87.8469	87.9459	88.0449	88.1439
178	88.7395	88.8390	88.9385	89.0380	89.1377
179	89.7366	89.8366	89.9366	90.0368	90.1370
180	90.7392	90.8398	90.9404	91.0411	91.1418
181	91.7474	91.8485	91.9497	92.0510	92.1523
182	92.7612	92.8629	92.9646	93.0664	93.1683

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
183	93.2702	93.3721	93.4741	93.5762	93.6783
184	94.2923	94.3948	94.4974	94.6000	94.7027
185	95.3200	95.4231	95.5262	95.6294	95.7326
186	96.3533	96.4569	96.5606	96.6643	96.7681
187	97.3921	97.4963	97.6005	97.7049	97.8092
188	98.4365	98.5413	98.6461	98.7509	98.8558
189	99.4865	99.5918	99.6972	99.8026	99.9081
190	100.5421	100.6479	100.7538	100.8598	100.9658
191	101.6032	101.7096	101.8161	101.9226	102.0292
192	102.6699	102.7769	102.8839	102.9910	103.0981
193	103.7421	103.8497	103.9573	104.0649	104.1726
194	104.8200	104.9281	105.0362	105.1444	105.2527
195	105.9034	106.0120	106.1207	106.229[illegible]	106.3383
196	106.9924	107.1016	107.2108	107.3201	107.4295
197	108.0869	108.1967	108.3065	108.4163	108.5263
198	109.1870	109.2973	109.4077	109.5181	109.6286
199	110.2927	110.4036	110.5145	110.6255	110.7365
200	111.4040	111.5154	111.6269	111.7384	111.8500
201	112.5208	112.6328	112.7448	112.8569	112.9691
202	113.6432	113.7557	113.8683	113.9810	114.0937
203	114.7711	114.8842	114.9974	115.1106	115.2239
204	115.9047	116.0183	116.1320	116.2458	116.3596
205	117.0438	117.1580	117.2723	117.3866	117.5010
206	118.1885	118.3032	118.4181	118.5329	118.6479
207	119.3387	119.4540	119.5694	119.6849	119.8004
208	120.4945	120.6104	120.7263	120.8423	120.9584
209	121.6559	121.7723	121.8888	122.0054	122.1220
210	122.8229	122.9399	123.0569	123.1740	123.2912
211	123.9954	124.1129	124.2306	124.3482	124.4660
212	125.1735	125.2916	125.4098	125.5280	125.6463
213	126.3572	126.4758	126.5946	126.7133	126.8322
214	127.5464	127.6656	127.7849	127.9042	128.0236
215	128.7412	128.8610	128.9808	129.1007	129 2207
216	129.9416				

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.5	.6	.7	.8	.9
183	93.7805	93.8828	93.9851	94.0874	94.1898
184	94.8055	94.9082	95.0111	95.1140	95.2170
185	95.8359	95.9393	96.0427	96.1462	96.2497
186	96.8720	96.9759	97.0799	97.1839	97.2880
187	97.9136	98.0181	98.1226	98.2272	98.3318
188	98.9608	99.0658	99.1709	99.2761	99.3813
189	100.0136	100.1192	100.2248	100.3305	100.4363
190	101.0719	101.1781	101.2843	101.3905	101.4968
191	102.1358	102.2425	102.3493	102.4561	102.5630
192	103.2053	103.3126	103.4199	103.5272	103.6347
193	104.2804	104.3882	104.4960	104.6040	104.7119
194	105.3610	105.4693	105.5778	105.6863	105.7948
195	106.4472	106.5561	106.6651	106.7741	106.8832
196	107.5389	107.6484	107.7579	107.8675	107.9772
197	108.6363	108.7463	108.8564	108.9665	109.0767
198	109.7392	109.8498	109.9604	110.0711	110.1819
199	110.8476	110.9588	111.0700	111.1813	111.2926
200	111.9617	112.0734	112.1851	112.2970	112.4088
201	113.0813	113.1935	113.3059	113.4182	113.5307
202	114.2065	114.3193	114.4322	114.5451	114.6581
203	115.3372	115.4506	115.5640	115.6775	115.7911
204	116.4735	116.5875	116.7015	116.8155	116.9296
205	117.6154	117.7299	117.8445	117.9591	118.0737
206	118.7629	118.8779	118.9930	119.1082	119.2234
207	119.9159	120.0315	120.1472	120.2629	120.3787
208	121.0745	121.1907	121.3069	121.4232	121.5395
209	122.2387	122.3554	122.4722	122.5890	122.7059
210	123.4084	123.5257	123.6430	123.7604	123.8779
211	124.5837	124.7016	124.8195	124.9374	125.0554
212	125.7646	125.8830	126.0015	126.1200	126.2385
213	126.9511	127.0700	127.1890	127.3081	127.4272
214	128.1431	128.2626	128.3822	128.5018	128.6215
215	129.3407	129.4608	129.5809	129.7011	129.8213

A TABLE *of the Areas of Circles in* WINE GALLONS, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 216 *Inches.*

Dia. in Inc.	0	.1	.2	.3	.4
1	0.0034	0.0041	0.0048	0.0057	0.0066
2	0.0136	0.0149	0.0164	0.0179	0.0195
3	0.0306	0.0326	0.0348	0.0370	0.0393
4	0.0544	0.0571	0.0599	0.0628	0.0658
5	0.0850	0.0884	0.0919	0.0955	0.0991
6	0.1224	0.1265	0.1306	0.1349	0.1392
7	0.1666	0.1713	0.1762	0.1811	0.1861
8	0.2176	0.2230	0.2286	0.2342	0.2399
9	0.2754	0.2815	0.2877	0.2940	0.3004
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4264	0.4341	0.4418
12	0.4896	0.4977	0.5060	0.5143	0.5227
13	0.5746	0.5834	0.5924	0.6014	0.6105
14	0.6664	0.6759	0.6855	0.6952	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8813	0.8922	0.9033	0.9144
17	0.9826	0.9941	1.0058	1.0175	1.0293
18	1.1016	1.1138	1.1262	1.1386	1.1511
19	1.2274	1.2403	1.2533	1.2664	1.2796
20	1.3600	1.3736	1.3873	1.4011	1.4149
21	1.4994	1.5137	1.5280	1.5425	1.5570
22	1.6456	1.6605	1.6756	1.6907	1.7059
23	1.7986	1.8142	1.8300	1.8458	1.8617
24	1.9584	1.9747	1.9911	2.0076	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3338	2.3517	2.3696
27	2.4786	2.4969	2.5154	2.5339	2.5525
28	2.6656	2.6846	2.7038	2.7230	2.7423
29	2.8594	2.8791	2.8989	2.9188	2.9388

A TABLE *of the Areas of Circles in* WINE GALLONS, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 216 *Inches.*

Dia. in Inc.	.5	.6	.7	.8	.9
1	0.0076	0.0087	0.0098	0.0110	0.0122
2	0.0212	0.0229	0.0247	0.0266	0.0285
3	0.0416	0.0440	0.0465	0.0490	0.0517
4	0.0688	0.0719	0.0751	0.0783	0.0816
5	0.1028	0.1066	0.1104	0.1143	0.1183
6	0.1436	0.1481	0.1526	0.1572	0.1618
7	0.1912	0.1963	0.2015	0.2068	0.2121
8	0.2456	0.2514	0.2573	0.2632	0.2693
9	0.3068	0.3133	0.3199	0.3265	0.3332
10	0.3748	0.3820	0.3892	0.3965	0.4039
11	0.4496	0.4575	0.4654	0.4734	0.4814
12	0.5312	0.5397	0.5483	0.5570	0.5657
13	0.6196	0.6288	0.6381	0.6474	0.6569
14	0.7148	0.7247	0.7347	0.7447	0.7548
15	0.8168	0.8274	0.8380	0.8487	0.8595
16	0.9256	0.9369	0.9482	0.9596	0.9710
17	1.0412	1.0531	1.0651	1.0772	1.0893
18	1.1636	1.1762	1.1889	1.2016	1.2145
19	1.2928	1.3061	1.3195	1.3329	1.3464
20	1.4288	1.4428	1.4568	1.4709	1.4851
21	1.5716	1.5863	1.6010	1.6158	1.6306
22	1.7212	1.7365	1.7519	1.7674	1.7829
23	1.8776	1.8936	1.9097	1.9258	1.9421
24	2.0408	2.0575	2.0743	2.0911	2.1080
25	2.2108	2.2282	2.2456	2.2631	2.2807
26	2.3876	2.4057	2.4238	2.4420	2.4602
27	2.5712	2.5899	2.6087	2.6276	2.6465
28	2.7616	2.7810	2,8005	2.8200	2.8397
29	2.9588	2.9789	2.9991	3.0193	3.0396

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	0	.1	.2	.3	.4
30	3.0600	3.0804	3.1009	3.1215	3.1421
31	3.2674	3.2885	3.3096	3.3309	3.3522
32	3.4816	3.5033	3.5252	3.5471	3.5691
33	3.7026	3.7250	3.7476	3.7702	3.7929
34	3.9304	3.9535	3.9767	4.0000	4.0234
35	4.1650	4.1888	4.2127	4.2367	4.2607
36	4.4064	4.4309	4.4554	4.4801	4.5048
37	4.6546	4.6797	4.7050	4.7303	4.7557
38	4.9096	4.9354	4.9614	4.9874	5.0135
39	5.1714	5.1979	5.2245	5.2512	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7712	5.7993	5.8274
42	5.9976	6.0261	6.0548	6.0835	6.1123
43	6.2866	6.3158	6.3452	6.3746	6.4041
44	6.5824	6.6123	6.6423	6.6724	6.7026
45	6.8850	6.9156	6.9463	6.9771	7.0079
46	7.1944	7.2257	7.2570	7.2885	7.3200
47	7.5106	7.5425	7.5746	7.6067	7.6389
48	7.8336	7.8662	7.8990	7.9318	7.9647
49	8.1634	8.1967	8.2301	8.2636	8.2972
50	8.5000	8.5340	8.5681	8.6023	8.6365
51	8.8434	8.8781	8.9128	8.9477	8.9826
52	9.1936	9.2289	9.2644	9.2999	9.3355
53	9.5506	9.5866	9.6228	9.6590	9.6953
54	9.9144	9.9511	9.9879	10.0248	10.0618
55	10.2850	10.3224	10.3599	10.3975	10.4351
56	10.6624	10.7005	10.7386	10.7769	10.8152
57	11.0466	11.0853	11.1242	11.1631	11.2021
58	11.4376	11.4770	11.5166	11.5562	11.5959
59	11.8354	11.8755	11.9157	11.9560	11.9964
60	12.2400	12.2808	12.3217	12.3627	12.4037

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	5.	.6	.7	.8	.9
30	3.1628	3.1836	3.2044	3.2253	3.2463
31	3.3736	3.3951	3.4166	3.4382	3.4598
32	3.5912	3.6133	3.6355	3.6578	3.6801
33	3.8156	3.8384	3.8613	3.8842	3.9073
34	4.0468	4.0703	4.0939	4.1175	4.1412
35	4.2848	4.3090	4.3332	4.3575	4.3819
36	4.5296	4.5545	4.5794	4.6044	4.6294
37	4.7812	4.8067	4.8323	4.8580	4.8837
38	5.0396	5.0658	5.0921	5.1184	5.1449
39	5.3048	5.3317	5.3587	5.3857	5.4128
40	5.5768	5.6044	5.6320	5.6597	5.6875
41	5.8556	5.8839	5.9122	5.9406	5.9690
42	6.1412	6.1701	6.1991	6.2282	6.2573
43	6.4336	6.4632	6.4929	6.5226	6.5525
44	6.7328	6.7631	6.7935	6.8239	6.8544
45	7.0388	7.0698	7.1008	7.1319	7.1631
46	7.3516	7.3833	7.4150	7.4468	7.4786
47	7.6712	7.7035	7.7359	7.7684	7.8009
48	7.9976	8.0306	8.0637	8.0968	8.1301
49	8.3308	8.3645	8.3983	8.4321	8.4660
50	8.6708	8.7052	8.7396	8.7741	8.8087
51	9.0176	9.0527	9.0878	9.1230	9.1582
52	9.3712	9.4069	9.4427	9.4786	9.5145
53	9.7316	9.7680	9.8045	9.8410	9.8777
54	10.0988	10.1359	10.1731	10.2103	10.2476
55	10.4728	10.5106	10.5484	10.5863	10.6243
56	10.8536	10.8921	10.9306	10.9692	11.0078
57	11.2412	11.2803	11.3195	11.3588	11.3981
58	11.6356	11.6754	11.7153	11.7552	11.7953
59	12.0368	12.0773	12.1179	12.1585	12.1992
60	12.4448	12.4860	12.5272	12.5685	12.6099

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
61	2.6514	12.6929	12.7344	12.7761	12.8178
62	13.0695	13.1117	13.1540	13.1963	13.2387
63	13.4945	13.5374	13.5804	13.6234	13.6665
64	13.9264	13.9699	14.0135	14.0572	14.1010
65	14.3650	14.4092	14.4535	14 4979	14.5423
66	14.8104	14.8553	14.9002	14.9453	14.9904
67	15.2626	15.3081	15.3538	15.3995	15.4453
68	15.7216	15.7678	15.8142	15.8606	15.9071
69	16.1874	16.2343	16.2813	16.3284	16.3756
70	16.6600	16.7076	16.7553	16.8031	16.8509
71	17.1394	17.1877	17.2360	17.2845	17.3330
72	17.6256	17.6745	17.7236	17.7727	17.8219
73	18.1186	18.1682	18.2180	18.2678	18.3177
74	18.6184	18.6687	18.7191	18.7696	18.8202
75	19.1250	19.1760	19.2271	19.2783	19.3295
76	19.6384	19.6901	19.7418	19.7937	19.8456
77	20.1586	20.2109	20.2634	20.3159	20.3685
78	20.6856	20.7386	20.7918	20.8450	20.8983
79	21.2194	21.2731	21.3269	21.3808	21.4348
80	21.7600	21.8144	21.8689	21.9235	21.9781
81	22.3074	22.3625	22.4176	22.4729	22.5282
82	22.8616	22.9173	22.9732	23.0291	23.0851
83	23.4226	23.4790	23.5356	23.5922	23.6489
84	23.9904	24.0475	24.1047	24.1620	24.2194
85	24.5650	24.6228	24.6807	24.7387	24.7967
86	25.1464	25.2049	25.2634	25.3221	25.3808
87	25.7346	25.7937	25.8530	25.9123	25.9717
88	26.3296	26.3894	26.4494	26.5094	26.5695
89	26.9314	26.9919	27.0525	27.1132	27.1740
90	27.5400	27.6012	27.6625	27.7239	27.7853
91	28.1554	28.2173	28.2792	28.3413	28.4034

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	·5	·6	·7	·8	·9
61	12.8596	12.9015	12.9434	12.9854	13.0274
62	13.2812	13.3237	13.3663	13.4090	13.4517
63	13.7096	13.7528	13.7961	13.8394	13.8829
64	14.1448	14.1887	14.2327	14.2767	14.3208
65	14.5868	14.6314	14.6760	14.7207	14.7655
66	15.0356	15.0809	15.1262	15.1716	15.2170
67	15.4912	15.5371	15.5831	15.6292	15.6753
68	15.9536	16.0002	16.0469	16.0936	16.1405
69	16.4228	16.4701	16.5175	16.5649	16.6124
70	16.8988	16.9468	16.9948	17.0429	17.0911
71	17.3816	17.4303	17.4790	17.5278	17.5766
72	17.8712	17.9205	17.9699	18.0194	18.0689
73	18.3676	18.4176	18.4677	18.5178	18.5681
74	18.8708	18.9215	18.9723	19.0231	19.0740
75	19.3808	19.4322	19.4836	19.5351	19.5867
76	19.8976	19.9497	20.0018	20.0540	20.1062
77	20.4212	20.4739	20.5267	20.5796	20.6325
78	20.9516	21.0050	21.0585	21.1120	21.1657
79	21.4888	21.5429	21.5971	21.6513	21.7056
80	22.0328	22.0876	22.1424	22.1973	22.2523
81	22.5836	22.6391	22.6946	22.7502	22.8058
82	23.1412	23.1973	23.2535	23.3098	23.3661
83	23.7056	23.7624	23.8193	23.8762	23.9333
84	24.2768	24.3343	24.3919	24.4495	24.5072
85	24.8548	24.9130	24.9712	25.0295	25.0879
86	25.4396	25.4985	25.5574	25.6164	25.6754
87	26.0312	26.0907	26.1503	26.2100	26.2697
88	26.6296	26.6898	26.7501	26.8104	26.8709
89	27.2348	27.2957	27.3567	27.4177	27.4788
90	27.8468	27.9084	27.9700	28.0317	28.0935
91	28.4656	28.5279	28.5902	28.6526	28.7150

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
92	28.7776	28.8401	28.9028	28.9655	29.0283
93	29.4066	29.4698	29.5332	29.5966	29.6601
94	30.0424	30.1063	30.1703	30.2344	30.2986
95	30.6850	30.7496	30.8143	30.8791	30.9439
96	31.3344	31.3997	31.4650	31.5305	31.5960
97	31.9906	32.0565	32.1226	32.1887	32.2549
98	32.6536	32.7202	32.7870	32.8538	32.9207
99	33.3234	33.3907	33.4581	33.5256	33.5932
100	34.0000	34.0680	34.1361	34.2043	34.2725
101	34.6834	34.7521	34.8208	34.8897	34.9586
102	35.3736	35.4429	35.5124	35.5819	35.6515
103	36.0706	36.1406	36.2108	36.2810	36.3513
104	36.7744	36.8451	36.9159	36.9868	37.0578
105	37.4850	37.5564	37.6279	37.6995	37.7711
106	38.2024	38.2745	38.3466	38.4189	38.4912
107	38.9266	38.9993	39.0722	39.1451	39.2181
108	39.6576	39.7310	39.8046	39.8782	39.9519
109	40.3954	40.4695	40.5437	40.6180	40.6924
110	41.1400	41.2148	41.2897	41.3647	41.4397
111	41.8914	41.9669	42.0424	42.1181	42.1938
112	42.6496	42.7257	42.8020	42.8783	42.9547
113	43.4146	43.4914	43.5684	43.6454	43.7225
114	44.1864	44.2639	44.3415	44.4192	44.4970
115	44.9650	45.0432	45.1215	45.1999	45.2783
116	45.7504	45.8293	45.9082	45.9873	46.0664
117	46.5426	46.6221	46.7018	46.7815	46.8613
118	47.3416	47.4218	47.5022	47.5826	47.6631
119	48.1474	48.2283	48.3093	48.3904	48.4716
120	48.9600	49.0416	49.1233	49.2051	49.2869

The

The Areas of Circles in Wine Gallons.

Dia. in Inc.	.5	.6	.7	.8	.9
92	29.0912	29.1541	29.2171	29.2802	29.3433
93	29.7236	29.7872	29.8509	29.9146	29.9785
94	30.3628	30.4271	30.4915	30.5559	30.6204
95	31.0088	31.0738	31.1388	31.2039	31.2691
96	31.6616	31.7273	31.7930	31.8588	31.9246
97	32.3212	32.3875	32.4539	32.5204	32.5869
98	32.9876	33.0546	33.1217	33.1888	33.2561
99	33.6608	33.7285	33.7963	33.8641	33.9320
100	34.3408	34.4092	34.4776	34.5461	34.6147
101	35.0276	35.0967	35.1658	35.2350	35.3042
102	35.7212	35.7909	35.8607	35.9306	36.0005
103	36.4216	36.4920	36.5625	36.6330	36.7037
104	37.1288	37.1999	37.2711	37.3423	37.4136
105	37.8428	37.9146	37.9864	38.0583	38.1303
106	38.5636	38.6361	38.7086	38.7812	38.8538
107	39.2912	39.3643	39.4375	39.5108	39.5841
108	40.0256	40.0994	40.1733	40.2472	40.3213
109	40.7668	40.8413	40.9159	40.9905	41.0652
110	41.5148	41.5900	41.6652	41.7405	41.8159
111	42.2696	42.3455	42.4214	42.4974	42.5734
112	43.0312	43.1077	43.1843	43.2610	43.3377
113	43.7996	43.8768	43.9541	44.0314	44.1089
114	44.5748	44.6527	44.7307	44.8087	44.8868
115	45.3568	45.4354	45.5140	45.5927	45.6715
116	46.1456	46.2249	46.3042	46.3836	46.4630
117	46.9412	47.0211	47.1011	47.1812	47.2613
118	47.7436	47.8242	47.9049	47.9856	48.0665
119	48.5528	48.6341	48.7155	48.7969	48.8784
120	49.3688	49.4508	49.5328	49.6149	49.6971

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	0	.1	.2	.3	.4
121	49.7794	49.8617	49.9440	50.0265	50.1090
122	50.6056	50.6885	50.7716	50.8547	50.9379
123	51.4386	51.5222	51.6060	51 6898	51.7737
124	52.2784	52.3627	52.4471	52.5316	52.6162
125	53.1250	53.2100	53.2951	53.3803	53 4655
126	53.9784	54.0641	54.1498	54.2357	54.3216
127	54.8386	54.9249	55 0114	55.0979	55.1845
128	55.7056	55.7926	55.8798	55.9670	56.0543
129	56.5794	56.6671	56.7549	56.8428	56.9308
130	57.4600	57.5484	57.6369	57.7255	57.8141
131	58.3474	58.4365	58.5256	58.6149	58.7042
132	59.2416	59.3313	59.4212	59.5111	59.6011
133	60.1426	60.2330	60.3236	60.4142	60.5049
134	61.0504	61.1415	61.2327	61.3240	61.4154
135	61.9650	62.0568	62.1487	62.2407	62.3327
136	62.8864	62.9789	63.0714	63.1641	63.2568
137	63.8146	63.9077	64.0010	64.0943	64.1877
138	64.7496	64.8434	64.9374	65.0314	65.1255
139	65.6914	65.7859	65.8805	65.9752	66.0700
140	66.6400	66.7352	66.8305	66.9259	67.0213
141	67.5954	67 6913	67.7872	67.8833	67.9794
142	68.5576	68.6541	68.7508	68 8475	68.9443
143	69.5266	69.6238	69.7212	69.8186	69.9161
144	70.5024	70.6003	70.6983	70.7964	70.8946
145	71.4850	71.5836	71.6823	71.7811	71.8799
146	72.4744	72.5737	72.6730	72.7725	72.8720
147	73.4706	73.5705	73.6706	73.7707	73.8709
148	74.4736	74.5742	74.6750	74.7758	74.8767
149	75.4834	75.5847	75.6861	75.7876	75.8892
150	76.5000	76.6020	76.7041	76.8063	76.9085
151	77.5234	77.6261	77.7288	77.8317	77.9346

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	·5	.6	·7	.8	·9
121	50.1916	50.2743	50.3570	50.4398	50.5226
122	51.0212	51.1045	51.1879	51.2714	51.3549
123	51.8576	51.9416	52.0257	52.1098	52.1941
124	52.7008	52.7855	52.8703	52.9551	53.0400
125	53.5508	53.6362	53.7216	53.8071	53.8927
126	54.4076	54.4937	54.5798	54.6660	54.7522
127	55.2712	55.3579	55.4447	55.5316	55.6185
128	56.1416	56.2290	56.3105	56.4[illegible]40	56.4917
129	57.0188	57.1069	57.1951	57.2833	57.3716
130	57.9028	57.9916	58.0804	58.1693	58.2583
131	58.7936	58.8831	58.9726	59.0622	59.1518
132	59.6912	59.7813	59.8715	59.9618	60.0521
133	60.5956	60.6864	60.7773	60.8682	60.9593
134	61.5068	61.5983	61.6899	61.7815	61.8732
135	62.4248	62.5170	62.6092	62.7015	62.7939
136	63.3496	63.4425	63.5354	63.6284	63.7214
137	64.2812	64.3747	64.4683	64.5620	64.6557
138	65.2196	65.3138	65.4081	65.5024	65.5969
139	66.1648	66.2597	66.3547	66.4497	66.5448
140	67.1168	67.2124	67.3080	67.4037	67.4995
141	68.0756	68.1719	68.2682	68.3646	68.4610
142	69.0412	69.1381	69.2351	69.3322	69.4293
143	70.0136	70.1112	70.2089	70.3066	70.4045
144	70.9928	71.0911	71.1895	71.2879	71.3864
145	71.9788	72.0778	72.1768	72.2759	72.3751
146	72.9716	73.0713	73.1710	73.2708	73.3706
147	73.9712	74.0715	74.1719	74.2724	74.3729
148	74.9776	75.0786	75.1797	75.2808	75.3821
149	75.9908	76.0925	76.1943	76.2961	76.3980
150	77.0108	77.1132	77.2156	77.3181	77.4207
151	78.0376	78.1407	78.2438	78.3470	78.4502

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	0.	.1	.2	.3	.4
152	78.5536	78.6569	78.7604	78.8639	78.9675
153	79.5906	79.6946	79.7988	79.9030	80.0073
154	80.6344	80.7391	80.8439	80.9488	81.0538
155	81.6850	81.7904	81.8959	82.0015	82.1071
156	82.7424	82.8485	82.9546	83.0609	83.1672
157	83.8066	83.9133	84.0202	84.1271	84.2341
158	84.8776	84.9850	85.0926	85.2002	85.3079
159	85.9554	86.0635	86.1717	86.2800	86.3884
160	87.0400	87.1488	87.2577	87.3667	87.4757
161	88.1314	88.2409	88.3504	88.4601	88.5698
162	89.2296	89.3397	89.4500	89.5603	89.6707
163	90.3346	90.4454	90.5564	90.6674	90.7785
164	91.4464	91.5579	91.6695	91.7812	91.8930
165	92.5650	92.6772	92.7895	92.9019	93.0143
166	93.6904	93.8033	93.9162	94.0293	94.1424
167	94.8226	94.9361	95.0498	95.1635	95.2773
168	95.9616	96.0758	96.1902	96.3046	96.4191
169	97.1074	97.2223	97.3373	97.4524	97.5676
170	98.2600	98.3756	98.4913	98.6071	98.7229
171	99.4194	99.5357	99.6520	99.7685	99.8850
172	100.5856	100.7025	100.8196	100.9367	101.0539
173	101.7586	101.8762	101.9940	102.1118	102.2297
174	102.9384	103.0567	103.1751	103.2936	103.4122
175	104.1250	104.2440	104.3631	104.4823	104.6015
176	105.3184	105.4381	105.5578	105.6777	105.7976
177	106.5186	106.6389	106.7594	106.8799	107.0005
178	107.7256	107.8466	107.9678	108.0890	108.2103
179	108.9394	109.0611	109.1829	109.3048	109.4268
180	110.1600	110.2824	110.4049	110.5275	110.6501
181	111.3874	111.5105	111.6336	111.7569	111.8802
182	112.6216	112.7453	112.8692	112.9931	113.1171

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	.5	.6	7.	.8	.9
152	79.0712	79.1749	79.2787	79.3826	79.4865
153	80.1116	80.2160	80.3205	80.4250	80.5297
154	81.1588	81.2639	81.3691	81.4743	81.5796
155	82.2128	82.3186	82.4244	82.5303	82.6363
156	83.2736	83.3801	83.4866	83.5932	83.6998
157	84.3412	84.4483	84.5555	84.6628	84.7701
158	85.4156	85.5234	85.6313	85.7392	85.8473
159	86.4968	86.6053	86.7139	86.8225	86.9312
160	87.5848	87.6940	87.8032	87.9125	88.0219
161	88.6796	88.7895	88.8994	89.0094	89.1194
162	89.7812	89.8917	90.0023	90.1130	90.2237
163	90.8896	91.0008	91.1121	91.2234	91.3349
164	92.0048	92.1167	92.2287	92.3407	92.4528
165	93.1268	93.2394	93.3520	93.4647	93.5775
166	94.2556	94.3689	94.4822	94.5956	94.7090
167	95.3912	95.5051	95.6191	95.7332	95.8473
168	96.5336	96.6482	96.7629	96.8776	96.9925
169	97.6828	97.7981	97.9135	98.0289	98.1444
170	98.8388	98.9548	99.0708	99.1869	99.3031
171	100.0016	100.1183	100.2350	100.3518	100.4686
172	101.1712	101.2885	101.4059	101.5234	101.6409
173	102.3476	102.4656	102.5837	102.7018	102.8201
174	103.5308	103.6495	103.7683	103.8871	104.0060
175	104.7208	104.8402	104.9596	105.0791	105.1987
176	105.9176	106.0377	106.1578	106.2780	106.3982
177	107.1212	107.2419	107.3627	107.4836	107.6045
178	108.3316	108.4530	108.5745	108.6960	108.8177
179	109.5488	109.6709	109.7931	109.9153	110.0376
180	110.7728	110.8956	111.0184	111.1413	111.2643
181	112.0036	112.1271	112.2506	112.3742	112.4978
182	113.2412	113.3653	113.4895	113.6138	113.7381

The

The Areas of Circles in Wine Gallons.

Dia in Inc.	.0	.1	.2	.3	.4
183	113.8620	113.9870	114.1116	114.2362	114.3609
184	115.1104	115.2355	115.3607	115.4860	115.6114
185	116.3650	116.4908	116.6167	116.7427	116.8687
186	117.6264	117.7529	117.8794	118.0061	118.1328
187	118.8946	119.0217	119.1490	119.2763	119.4037
188	120.1696	120.2974	120.4254	120.5534	120.6815
189	121.4514	121.5799	121.7085	121.8372	121.9660
190	122.7400	122.8692	122.9985	123.1279	123.2573
191	124.0354	124.1653	124.2952	124.4253	124.5554
192	125.3376	125.4681	125.5988	125.7295	125.8603
193	126.6466	126.7778	126.9092	127.0406	127.1721
194	127.9624	128.0943	128.2263	128.3584	128.4906
195	129.2850	129.4176	129.5503	129.6831	129.8159
196	130.6144	130.7477	130.8810	131.0145	131.1480
197	131.9506	132.0845	132.2186	132.3527	132.4869
198	133.2936	133.4282	133.5630	133.6978	133.8327
199	134.6434	134.7787	134.9141	135.0496	135.1852
200	136.0000	136.1360	136.2721	136.4083	136.5445
201	137.3634	137.5001	137.6368	137.7737	137.9106
202	138.7336	138.8709	139.0084	139.1459	139.2835
203	140.1106	140.2486	140.3868	140.5250	140.6633
204	141.4944	141.6331	141.7719	141.9108	142.0498
205	142.8850	143.0244	143.1639	143.3035	143.4431
206	144.2824	144.4225	144.5626	144.7029	144.8432
207	145.6866	145.8273	145.9682	146.1091	146.2501
208	147.0976	147.2390	147.3806	147.5222	147.6639
209	148.5154	148.6575	148.7997	148.9420	149.0844
210	149.9400	150.0828	150.2257	150.3687	150.5117
211	151.3714	151.5149	151.6584	151.8021	151.9458
212	152.8096	152.9537	153.0980	153.2423	153.3867
213	154.2546	154.3994	154.5444	154.6894	154.8345
214	155.7064	155.8519	155.9975	156.1432	156.2890
215	157.1650	157.3112	157.4575	157.6039	157.7503
216	158.6304				

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	.5	.6	.7	.8	.9
183	114.4856	114.6104	114.7353	114.8602	114.9853
184	115.7368	115.8623	115.9879	116.1135	116.2392
185	116.9948	117.1210	117.2472	117.3735	117.4999
186	118.2596	118.3865	118.5134	118.6404	118.7674
187	119.5312	119.6587	119.7863	119.9140	120.0417
188	120.8096	120.9378	121.0661	121.1944	121.3229
189	122.0948	122.2237	122.3527	122.4817	122.6108
190	123.3868	123.5164	123.6460	123.7757	123.9055
191	124.6856	124.8159	124.9462	125.0766	125.2070
192	125.9912	126.1221	126.2531	126.3842	126.5153
193	127.3036	127.4352	127.5669	127.6986	127.8305
194	128.6228	128.7551	128.8875	129.0199	129.1524
195	129.9488	130.0818	130.2148	130.3479	130.4811
196	131.2816	131.4153	131.5490	131.6828	131.8166
197	132.6212	132.7555	132.8899	133.0244	133.1589
198	133.9676	134.1026	134.2377	134.3728	134.5081
199	135.3208	135.4565	135.5923	135.7281	135.8640
200	136.6808	136.8172	136.9536	137.0901	137.2267
201	138.0476	138.1847	138.3218	138.4590	138.5962
202	139.4212	139.5589	139.6967	139.8346	139.9725
203	140.8016	140.9400	141.0785	141.2170	141.3557
204	142.1888	142.3279	142.4671	142.6063	142.7456
205	143.5828	143.7226	143.8624	144.0023	144.1423
206	144.9836	145.1241	145.2646	145.4052	145.5458
207	146.3912	146.5323	146.6735	146.8148	146.9561
208	147.8056	147.9474	148.0893	148.2312	148.3733
209	149.2268	149.3693	149.5119	149.6545	149.7972
210	150.6548	150.7980	150.9412	151.0845	151.2279
211	152.0896	152.2335	152.3774	152.5214	152.6654
212	153.5312	153.6757	153.8203	153.9650	154.1097
213	154.9796	155.1248	155.2701	155.4154	155.5609
214	156.4348	156.5807	156.7267	156.8727	157.0188
215	157.8968	158.0434	158.1900	158.3367	158.4835

The Uſes of the preceding Tables of Areas are ſo very obvious, that we apprehend one Example will be ſufficient to illuſtrate them both. Let that be in finding the Area of a Circle, in Ale and Wine Gallons, whoſe Diameter is 45.4 Inches.

Againſt 45 in the 1ſt Column, under the Words *Diam. in Inches*, and in the 6th Column under .4, we have 5.7405 Gallons in the Table of Ale Areas; and 7.0079 Gallons, in the Table of Wine Areas, the Anſwer ſought.

Since the preceding Tables of Areas are computed by Methods more exact and eaſy than any I have yet ſeen; it may perhaps not be improper to give them here.

Both theſe Tables are formed by Addition only, from two common *Addends*; *i. e.* from .000055702 for Ale,* and .000068 for Wine Gallons.—Thus, the common Increaſe of the Diameters, according to the Tables, being $\frac{1}{10}$th of an Inch, the Ale Area of any propoſed Diameter muſt, in Order to obtain the ſucceeding Area, be increaſed by .000055702 of a Gallon, more than the Increaſe of the propoſed Area from the preceding One: And, in the Table of Wine Areas, this Increaſe, or common Addend, is .000068 of a Gallon.

But,

* The Reaſon of 000055702 being a common Addend for Ale, and .000068 a common Addend for Wine Gallons, is very evident from the *Lemma* in *Pa.* 154: For, in this Caſe, $n = .1$; therefore $2n^2 = \overline{.1}|^2 \times 2$ (or .02), which being multiplied by .0027851 for Ale, and .0034 for Wine (in Order to have its Meaſure in Parts of a Gallon), gives .000055702, the common Addend for Ale, and .000068, the common Addend for Wine Gallons, when the Diameter of the Circle is conſtantly increaſed by one-tenth of an Inch.

But, to be more explicit, it may be proper to ſhew the Method of Continuation of the Tables; each, for Inſtance, from 120 Inches Diameter: Which Method, for Ale Gallons, is as follows.

To .06681 4549 (being the Sum of .000027851, and 1199 times the common Addend) add the common Addend .000055702; and the Sum .066870251 (called the *reſerved Sum)* being added to 40.10544, the Area for 120 Inches Diameter, gives 40.172310251 for the Area of 120.1 Inches Diameter: Again, to .066870251, the reſerved Sum, add the common Addend .000055702; and this *(reſerved)* Sum *(i. e.* .066925953) being added to 40.172310251, gives 40.239236204 for the Area of a Circle in Ale Gallons, whoſe Diameter is 120.2 Inches. Proceed in the ſame Manner, ſtill adding the laſt *reſerved Sum* and the *common Addend* (.000055702) together, and then adding the Sum of thoſe two to the laſt Area, gives the ſucceeding Area; *i. e.* when the Diameter is increaſed by $\frac{1}{10}$th of an Inch.

And for the Table of Wine Areas (which is derived from the very ſame Principle), proceed, from 120 Inches Diameter, thus.

To .081566 (being the Sum of .000034, and 1199 times the common Addend) add the common Addend .000068; and this Sum .081634 (called the *reſerved Sum)* being added to 48.96, the Area for 120 Inches Diameter, gives 49.041634 for 120.1 Inches: Again, to .081634, the laſt reſerved Sum, add the common Addend .000068, and we ſhall then get .081702 for the reſerved Sum; which being added to 49.041634 (found above) gives 49.123336, the Area for 120.2 Inches Diameter; and ſo on: See the following Operation.

Inches.	Gallons.		
120 - -	48.960600	.081566	*reserved* Sum at
Add	.081634	.000068	[120 Inches.
120.1 -	49.041634	.081634	*reserved.*
Add	.081702	.000068	
120.2 -	49.123336	.081702	*reserved.*
Add	.081770	.000068	
120.3 -	49.205106	.081770	
	.081838	.000068	
120.4 -	49.286944	.081838	
	.081906	.000068	
120.5 -	49.368850	.081906	
	.081974	.000068	
120.6 -	49.450824	.081974	
	&c.	*&c.*	

A TABLE

☞ In the 1st L. of the Errata, the Correction should have been *in the single.*

A TABLE *of the* Areas of the Segments of a Circle *whose Diameter is Unity, and supposed to be divided into* 1000 *equal Parts.*

Versed Sine	*Seg. Area*	Versed Sine	*Seg. Area*	Versed Sine	*Seg. Area*
.001	.000042	.030	.006865	.059	.018766
.002	.000119	.031	.007209	.060	.019239
.003	.000219	.032	.007558	.061	.019716
.004	.000337	.033	.007913	.062	.020196
.005	.000470	.034	.008273	.063	.020680
.006	.000618	.035	.008638	.064	.021168
.007	.000779	.036	.009008	.065	.021659
.008	.000951	.037	.009383	.066	.022154
.009	.001135	.038	.009763	.067	.022652
.010	.001329	.039	.010148	.068	.023154
.011	.001533	.040	.010537	.069	.023659
.012	.001746	.041	.010931	.070	.024168
.013	.001968	.042	.011330	.071	.024680
.014	.002199	.043	.011734	.072	.025195
.015	.002438	.044	.012142	.073	.025714
.016	.002685	.045	.012554	.074	.026236
.017	.002940	.046	012971	.075	.026761
.018	.003202	.047	.013392	.076	.027289
.019	.003471	.048	.013818	.077	.027821
.020	.003748	.049	.014247	.078	.028356
.021	.004031	.050	.014681	.079	.028894
.022	.004322	.051	.015119	.080	.029435
.023	.004618	.052	.015561	.081	.029979
.024	.004921	.053	.016007	.082	.030526
.025	.005230	.054	.016457	.083	.031076
.026	.005546	.055	.016911	.084	.031629
.027	.005867	.056	.017369	.085	.032186
.028	.006194	.057	.017831	.086	.032745
.029	.006527	.058	.018296	.087	.033307

The

The Areas of the Segments of a Circle.

Versed Sine	*Seg. Area*	Versed Sine	*Seg. Area*	Versed Sine	*Seg. Area*
.088	.033872	.119	.052736	.150	.073874
.089	.034441	.120	.053385	.151	.074589
.090	.035011	.121	.054036	.152	.075306
.091	.035585	.122	.054689	.153	.076026
.092	.036162	.123	.055345	.154	.076747
.093	.036741	.124	.056003	.155	.077469
.094	.037323	.125	.056663	.156	.078194
.095	.037909	.126	.057326	.157	.078921
.096	.038496	.127	.057991	.158	.079649
.097	.039087	.128	.058658	.159	.080380
.098	.039680	.129	.059327	.160	.081112
.099	.040276	.130	.059999	.161	.081846
.100	.040875	.131	.060672	.162	.082582
.101	.041476	.132	.061348	.163	.083320
.102	.042080	.133	.062026	.164	.084059
.103	.042687	.134	.062707	.165	.084801
.104	.043296	.135	.063389	.166	.085544
.105	.043908	.136	.064074	.167	.086289
.106	.044522	.137	.064760	.168	.087036
.107	.045139	.138	.065449	.169	.087785
.108	.045759	.139	.066140	.170	.088535
.109	.046381	.140	.066833	.171	.089287
.110	.047005	.141	.067528	.172	.090041
.111	.047632	.142	.068225	.173	.090797
.112	.048262	.143	.068924	.174	.091554
.113	.048894	.144	.069625	.175	.092313
.114	.049528	.145	.070328	.176	.093074
.115	.050165	.146	.071033	.177	.093836
.116	.050804	.147	.071741	.178	.094601
.117	.051446	.148	.072450	.179	.095366
.118	.052090	.149	.073161	.180	.096134

The

The Areas of the Segments of a Circle.

Versed Sine	Seg. Area	Versed Sine	Seg. Area	Versed Sine	Seg. Area
.181	.096903	.212	.121529	.243	.147512
.182	.097674	.213	.122347	.244	.148371
.183	.098447	.214	.123167	.245	.149230
.184	.099221	.215	.123988	.246	.150091
.185	.099997	.216	.124810	.247	.150953
.186	.100774	.217	.125634	.248	.151816
.187	.101553	.218	.126459	.249	.152680
.188	.102334	.219	.127285	.250	.153546
.189	.103116	.220	.128113	.251	.154412
.190	.103900	.221	.128942	.252	.155280
.191	.104685	.222	.129773	.253	.156149
.192	.105472	.223	.130605	.254	.157019
.193	.106261	.224	.131438	.255	.157890
.194	.107051	.225	.132272	.256	.158762
.195	.107842	.226	.133108	.257	.159636
.196	.108636	.227	.133945	.258	.160510
.197	.109430	.228	.134784	.259	.161386
.198	.110226	.229	.135624	.260	.162263
.199	.111024	.230	.136465	.261	.163140
.200	.111823	.231	.137307	.262	.164019
.201	.112624	.232	.138150	.263	.164899
.202	.113426	.233	.138995	.264	.165780
.203	.114230	.234	.139841	.265	.166663
.204	.115035	.235	.140688	.266	.167546
.205	.115842	.236	.141537	.267	.168430
.206	.116650	.237	.142387	.268	.169315
.207	.117460	.238	.143238	.269	.170202
.208	.118271	.239	.144091	.270	.171089
.209	.119083	.240	.144944	.271	.171978
.210	.119897	.241	.145799	.272	.172867
.211	.120712	.242	.146655	.273	.173758

The

The Areas of the Segments of a Circle.

Versed Sine	Seg. Area	Versed Sine	Seg. Area	Versed Sine	Seg. Area
.274	.174649	.305	.202761	.336	.231689
.275	.175542	.306	.203683	.337	.232634
.276	.176435	.307	.204605	.338	.233580
.277	.177330	.308	.205527	.339	.234526
.278	.178225	.309	.206451	.340	.235473
.279	.179122	.310	.207376	.341	.236421
.280	.180019	.311	.208301	.342	.237369
.281	.180918	.312	.209227	.343	.238318
.282	.181817	.313	.210154	.344	.239268
.283	.182718	.314	.211082	.345	.240218
.284	.183619	.315	.212011	.346	.241169
.285	.184521	.316	.212940	.347	.242121
.286	.185425	.317	.213871	.348	.243074
.287	.186329	.318	.214802	.349	.244026
.288	.187234	.319	.215733	.350	.244980
.289	.188140	.320	.216666	.351	.245934
.290	.189047	.321	.217599	.352	.246889
.291	.189955	.322	.218533	.353	.247845
.292	.190864	.323	.219468	.354	.248801
.293	.191775	.324	.220404	.355	.249757
294	.192684	.325	.221340	.356	.250715
.295	.193596	.326	.222277	.357	.251673
.296	.194509	.327	.223215	.358	.252631
.297	.195422	.328	.224154	.359	.253590
.298	.196337	.329	.225093	.360	.254550
.299	.197252	.330	.226033	.361	.255510
.300	.198168	.331	.226974	.362	.256471
.301	.199085	.332	.227915	.363	.257433
.302	.200003	.333	.228858	.364	.258395
.303	.200922	.334	.229801	.365	.259357
.304	.201847	.335	.230745	.366	.260320

The

The Areas of the Segments of a Circle.

Versed Sine	Seg. Area	Versed Sine	Seg. Area	Versed Sine	Seg. Area
.367	.261284	.396	.289453	.425	.317981
.368	.262248	.397	.290432	.426	.318970
.369	.263213	.398	.291411	.427	.319959
.370	.264178	.399	.292390	.428	.320948
.371	.265144	.400	.293369	.429	.321938
.372	.266111	.401	.294349	.430	.322928
.373	.267078	.402	.295330	.431	.323918
.374	.268045	.403	.296311	.432	.324909
.375	.269013	.404	.297292	.433	.325900
.376	.269982	.405	.298273	.434	.326892
.377	.270951	.406	.299255	.435	.327882
.378	.271920	.407	.300238	.436	.328874
.379	.272890	.408	.301220	.437	.329866
.380	.273861	.409	.302203	.438	.330858
.381	.274832	.410	.303187	.439	.331850
.382	.275803	.411	.304171	.440	.332843
.383	.276775	.412	.305155	.441	.333836
.384	.277748	.413	.306140	.442	.334829
.385	.278721	.414	.307125	.443	.335822
.386	.279694	.415	.308110	.444	.336816
.387	.280668	.416	.309095	.445	.337810
.388	.281642	.417	.310081	.446	.338804
.389	.282617	.418	.311068	.447	.339798
.390	.283592	.419	.312054	.448	340793
.391	.284568	.420	.313041	.449	.341787
.392	.285544	.421	.314029	.450	.342782
.393	.286521	.422	.315016	.451	.343777
.394	.287498	.423	.316004	.452	.344772
.395	.288476	.424	.316992	.453	.345768

The Areas of the Segments of a Circle.

Versed Sine	Seg. Area	Versed Sine	Seg. Area
.454	.346764	.478	.370706
.455	.347759	.479	.371705
.456	.348755	.480	.372704
.457	.349752	.481	.373703
.458	.350748	.482	.374702
.459	.351745	.483	.375702
.460	.352742	.484	.376702
.461	.353739	.485	.377701
.462	.354736	.486	.378701
.463	.355732	.487	.379700
.464	.356730	.488	.380700
.465	.357727	.489	.381699
.466	.358725	.490	.382699
.467	.359723	.491	.383699
.468	.360721	.492	.384699
.469	.361719	.493	.385699
.470	.362717	.494	.386699
.471	.363715	.495	.387699
.472	.364713	.496	.388699
.473	.365712	.497	.389699
.474	.366710	.498	.390699
.475	.367709	.499	.391699
.476	.368708	.500	.392699
.477	.369707		

The

The Use of the Line M.D *on the Sliding-Rule, in* Malt-Gauging.

It is presumed that no great Difficulty can arise in the Practice of *Malt-Gauging*, if what has been delivered in *Sect.* VII. and VIII. be duly attended to: For it is well known, that a Malster's *Cistern* (*i. e.* where the Barley is steeped) and also the *Couch* (*i. e.* where it is laid after it has been steeped) are chiefly in the Form of a *Cylinder*, the *Frustum* of a *Cone*, or a *rectangular Parallelopipedon*; which Figures, with Variety of others, have been fully treated of in the above-mentioned *Sections*, and their Contents computed, in Malt Bushels, both by Pen and Sliding-Rule: However, it may not be amiss to give a few Propositions, and exemplify the same, in Order to shew the Use of the Line M.D (commonly called *Malt-Depth*) on the Sliding Rule.

Note. When the Barley is taken out of the Couch and spread on the Floor, it is then called a *Floor of Malt*.

PROP. I.

The Length and Breadth of a rectangular Parallelogram being given in Inches; to find the Area thereof (in Inches) *by the Line* M.D, &c. *on the Sliding-Rule.*

RULE.

To either of the given *Dimensions* on M.D, set the *other* on the Line B, or, which is the very same (on some Rules), that marked N; then

against 1 (*viz.* Unity) on M.D, is the required Area on the Slide.

EXAMPLE.

Let the Length of a rectangular Parallelogram be 12, and the Breadth 7 Inches; required the Area thereof.

To 7 on M.D, set 12 on B (or N); then opposite 1 on M.D, is 84 on B, the Area sought.*

It is to be observed, in Examples of this Kind, that 1 on M.D must always represent Unity, and therefore the Factor taken on that Line, if greater than 21.5042, must be divided by such a Power of 10 as will cause 1 on M.D to denote Unity; then the Number opposite thereto being multiplied by the same Power of 10 as the abovementioned Factor was divided by, and the Product will be the Answer sought.

PROP. II.

The Length and Breadth of a rectangular Parallelogram being given in Inches; to find its Area in Malt Bushels, by the Line M.D.

RULE.

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* It is manifest, that, by setting 12 on B (or N) to 7 on M.D (or 7 on B to 12 on M.D), we shall obtain (on B) the Sum of the Distances of 1 to 12 on B, and 1 to 7 on M.D (or 1 to 7 on B, and 1 to 12 on M.D): But these Distances (by the Construction of the Lines) are as the Logarithms of 7 and 12 respectively; consequently the Sum of those Distances will be as the Sum of the Logarithms of those Numbers, which, by the Property of Logarithms, is as the Logarithm of their Product.

Rule.

To either of the given Dimensions on M.D, ſet the *other* on B (or N); then againſt 1 (*i. e.* Unity) on A, is the required Area on B.

Example.

Suppoſe the Length of a rectangular Ciſtern is 180, and the Breadth 53 5 Inches; required the Area thereof in Malt Buſhels.

Note. As it is ſometimes difficult to eſtimate the *true* Value of the Number found upon the Line B; it may therefore be proper to lay down the following Directions.

Let 1, near the Middle of M.D, denote *Unity*; and the Number oppoſite thereto, at the Braſs Pin on A, repreſent 2150.42; then, in Order to have 1 at the Middle of the Line A to ſtand for *Unity* (inſtead of 1000), we need but to conceive the Product of the two given Factors to be divided by 1000:† Thus, in the Example before us, to 1.8 (inſtead

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† By ſuppoſing the Product of the two given Factors to be divided by 1000, is the very ſame Thing as ſuppoſing three Radii taken from the Lines M.D and B: For by ſetting 5.35 (inſtead of 53.5) on B to 1.8 (inſtead of 180) we ſhall obtain the Diſtance of 1 to 1.8 on M.D, and of 1 to 5.35 on B, in one Sum on B; which Diſtance is diminiſhed by that of 1 to 2.15042 (*i. e.* $\frac{2150.42}{1000}$) on A: Moreover, by the Conſtruction of the Lines, theſe Diſtances are as the Logarithms of the Numbers 1.8, 5.35, and 2.15042 reſpectively; whence, by the Properties of Logarithms, the *Log.* 1.8 + L. 5.35 — L. 2 15042 (= L. 180 + L. 53.5 — L. 2150.42 = L. 4.479) $= \frac{1.8 \times 5.35}{2.15942}$ $(= \frac{180 \times 53.5}{2150.42})$ = 4.479.

(instead of 180) on M.D, set 5.35 (instead of 53.5) on B; then against 1 on A is 4.5 Bushels, *nearly*, on B.

Note. The Answer will come out the very same as above; if 1 at the Beginning of the Line A denotes *Unity*, the Number at the Brass Pin, opposite 1 (*viz*. Unity) on M.D 215.042, and the Product of the two Factors on M.D and B be supposed to be divided by 100 (instead of 1000); but it will, I presume, be better to keep to one *general Method*, as given above.

If the given Length of the Cistern is not less than 100 nor greater than 10000 (which last indeed never happens in Practice); then the required Area may be obtained, with more Ease to a Learner, by the Lines A and B. — Thus, in the last Example, to 2150 on A, set 53.5 on B; then opposite 180 (on the 1st Radius) on A, is 4.5 Bushels, *nearly*, on B.

Prop. III.

The Length, Breadth, and Depth of a rectangular Parallelopipedon being given; to find its Content in Malt Bushels, by the Line M.D, &c. *on the Sliding-Rule.*

Rule.

To any of the three given Dimensions on M.D, set either of the other two on B (N); then against the third Dimension on A, is the required Content on B.

Example.

EXAMPLE.

Let the Length of a rectangular Floor of Malt be 350, the Breadth 160, and the Depth 6.5 Inches; required its Content in Malt Bushels.

To 350 (or rather 3.5) on M.D, set 160 (or 16) on B *(vid.* the last *Example)*; then against 6.5 (on the 2d Radius) on A, is 169 Bushels on B.

Note. As 1 in the Middle of the Line A, according to our Method of Estimation, always denotes *Unity*, the third Dimension, when it exceeds 10, cannot be found on A: It will therefore be necessary, in such Cases, to have Recourse to the Method laid down in *Pa.* 42.

Thus, for Instance, suppose the last Example had been a rectangular Cistern, whose Depth had been 65 Inches, and the other Dimensions the same as before.

Then, the Rule being set as above, against 6.5 *(i. e.* $\frac{1}{10}$th of 65) we have 169; which being multiplied by 10, gives 1690 Bushels, the required Content of the Cistern, *nearly*.

The END.

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